

Un modèle particulière pour une équation d'interface

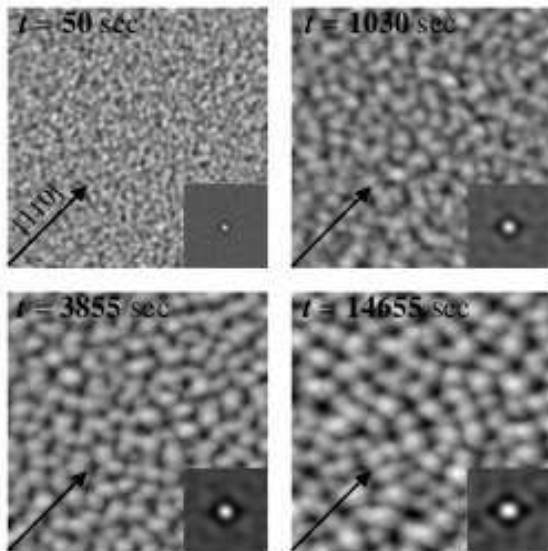
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¹MAPMO

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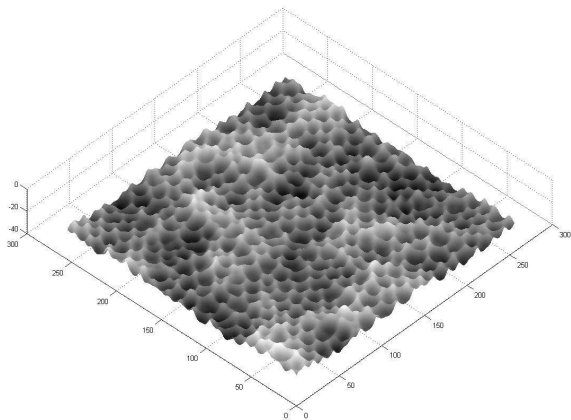
15^e journée CaSciModOT – Orléans – 13-12-11

Expérience plane (érosion par pulvérisation du Palladium)



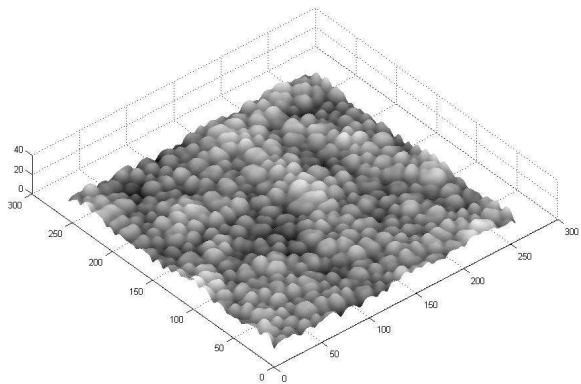
Simulations

érosion



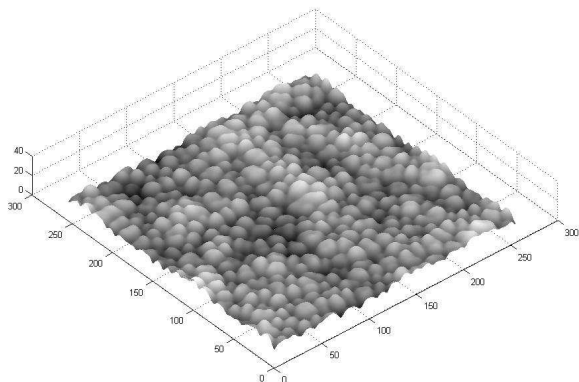
Simulations

croissance



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croissance



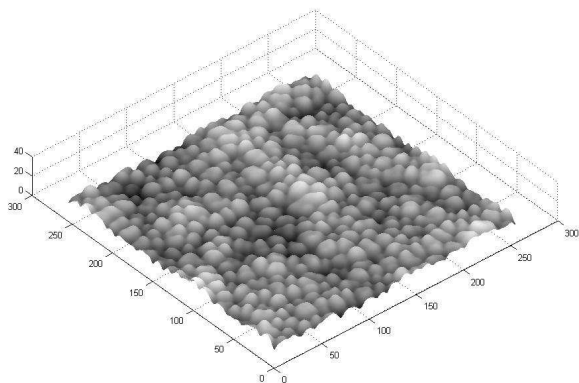
$$\partial_t h - \frac{\lambda}{2} (\nabla_x h)^2 = -\nu \nabla_x^2 h - K \nabla_x^4 h, \quad x \in [0, L]$$

$\nu > 0$: anti-diffusion

$K > 0$: diffusion de surface

Simulations

croissance : $\lambda > 0$



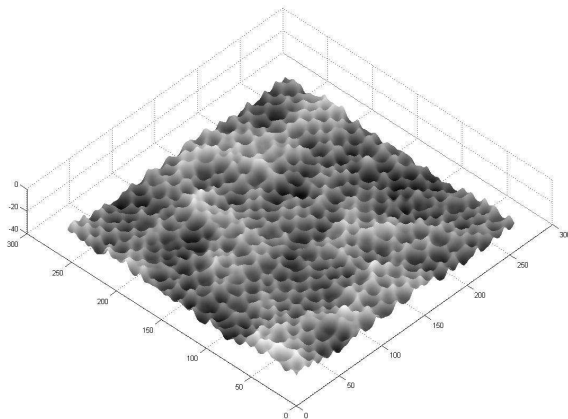
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Simulations

érosion : $\lambda < 0$

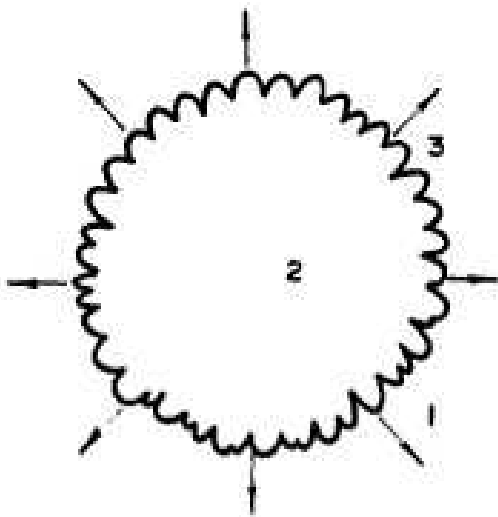


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$\nu > 0$: anti-diffusion

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Front de flamme (Sivashinsky)



1 dimension périodique

$$\partial_t h - \frac{\lambda}{2} (\partial_x h)^2 = -\nu \partial_{xx} h - K \partial_{xxxx} h, \quad x \in [0, L]$$

“Vitesse” $u = -\partial_x h$

$$\partial_t u - \frac{\lambda}{2} u \partial_x u = -\nu \partial_{xx} u - K \partial_{xxxx} u, \quad u(t, 0) = u(t, L).$$

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Burgers non visqueux

$$\partial_t u + u \partial_x u = 0$$

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Burgers visqueux

$$\partial_t u + u \partial_x u = +\nu \partial_{xx} u$$

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$$\partial_t u - \frac{\lambda}{2} u \partial_x u = -\nu \partial_{xx} u - K \partial_{xxxx} u, \quad u(t, 0) = u(t, L).$$

anti-diffusion (instable)

$$\partial_t u + u \partial_x u = -\nu \partial_{xx} u$$

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$$\partial_t h - \frac{\lambda}{2} (\partial_x h)^2 = -\nu \partial_{xx} h - K \partial_{xxxx} h, \quad x \in [0, L]$$

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K-S

$$\partial_t u + u \partial_x u = -\nu \partial_{xx} u - K \partial_{xxxx} u$$

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K-S

$$\partial_t u + u \partial_x u = -\nu \partial_{xx} u - K \partial_{xxxx} u$$

Équilibre entre $-\nu \partial_{xx} u$ et $-K \partial_{xxxx} u$

\implies Apparition de **structures** de taille $\ell_c = 2\pi \sqrt{\frac{2K}{\nu}}$.

La **modélisation** sans mathématiques, c'est comme

un baiser sans sel (PB)

La **modélisation** sans mathématiques, c'est comme

un baiser sans sel (PB)

ou une soupe sans moustache (JLR)

Encart gracieusement offert par la FDP (GB)

Modèle particulière

$$\begin{aligned}x_i'(t) &= v_i(t) \\v_i'(t) &= \alpha(v_{i-1}(t) - 2v_i(t) + v_{i+1}(t)) \\&\quad + \beta(v_{i-2}(t) - 2v_i(t) + v_{i+2}(t))\end{aligned}$$

$\alpha > 0$ attraction

$\beta < 0$ répulsion

$$\alpha(v_{i-1}(t) - 2v_i(t) + v_{i+1}(t)) \approx \alpha \partial_{xx} u$$

$$\beta(v_{i-2}(t) - 2v_i(t) + v_{i+2}(t))$$

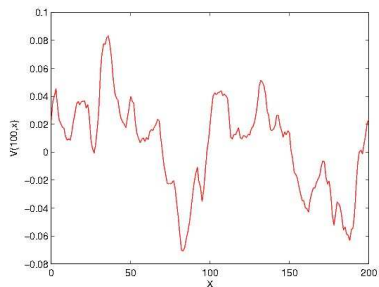
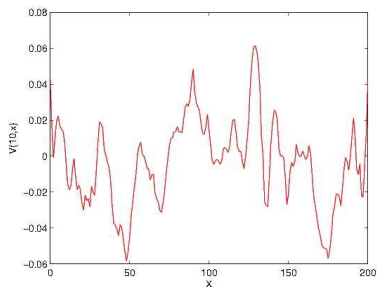
$$= \beta(v_{i-2}(t) - 6v_{i-1}(t) + 4v_i(t) - 6v_{i+1}(t) + v_{i+2}(t))$$

$$+ 4\beta(v_{i-1}(t) - 2v_i(t) + v_{i+1}(t))$$

$$\approx \beta \partial_{xxxx} u + 4\beta \partial_{xx} u$$

Vitesses

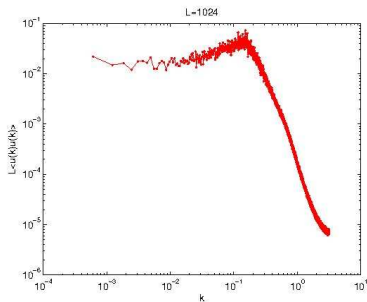
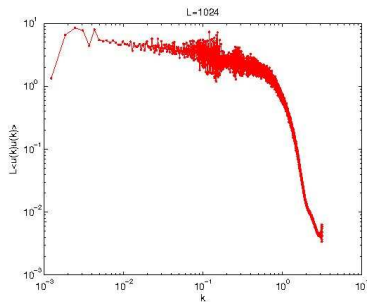
Profils de vitesses (moyenne d'ensembles)



$\alpha = 3.5$, $T = 10$ (gauche), $T = 100$ (droite)

Spectres

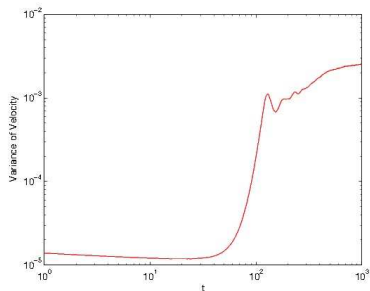
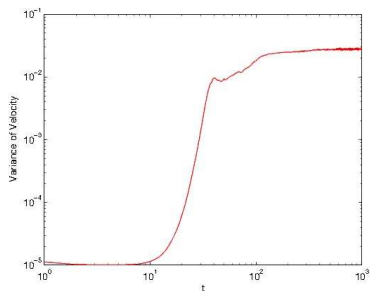
Spectre de puissance des vitesses



$\alpha = 3$ (gauche), $\alpha = 3.5$ (droite)

Évolution de la rugosité

Indicateur de rugosité $\langle (v(t) - \bar{v}(t))^2 \rangle$, avec $\bar{v}(t) = \frac{1}{N} \sum v_i(t)$



$\alpha = 3$ (gauche), $\alpha = 3.5$ (droite)

That's all, folks !