Numerical modeling of the convection and degassing of a magma system: the case of the Erebus lava lake, Antarctica

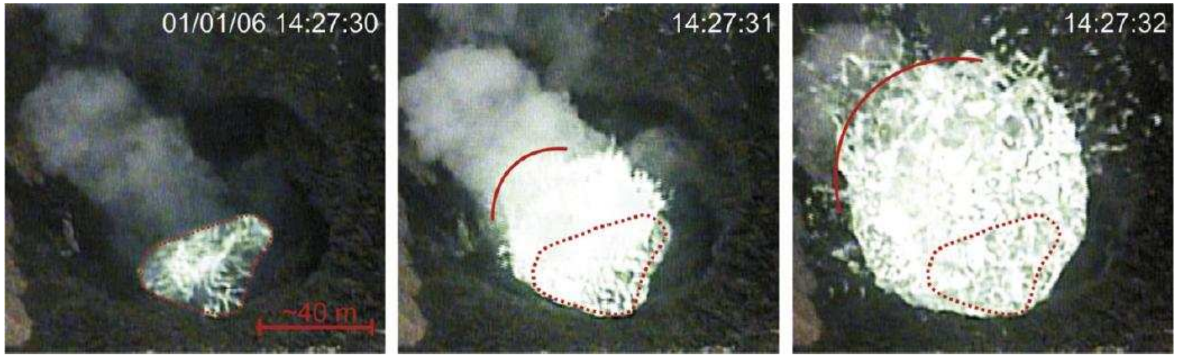
Indira MOLINA
Under direction of Alain BURGISSER

General objective:
Can we simulate the permanent convection at Erebus?

1. Introduction and objectives
2. Numerical modeling
   a) The role of crystals
   b) The role of bubbles
3. General conclusions

- Phonolitic lava lake
- Steady state (e.g., *Kyle, 1977; Kelly et al, 2008, Oppenheimer et al, 2009*)
- Window of the magmatic system

Explosive degassing (strombolian)

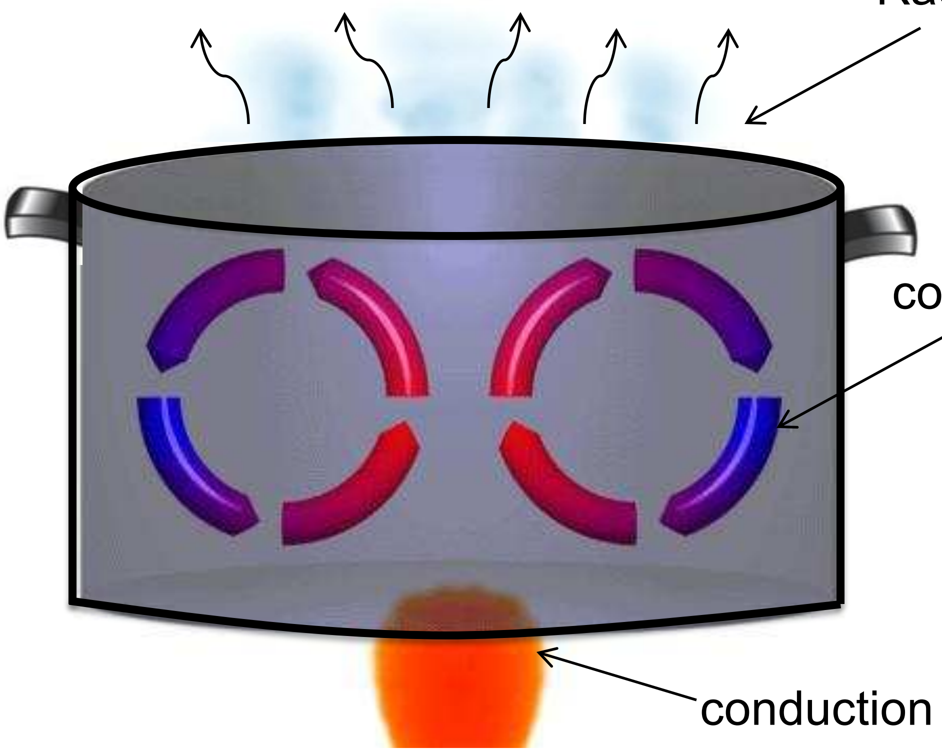


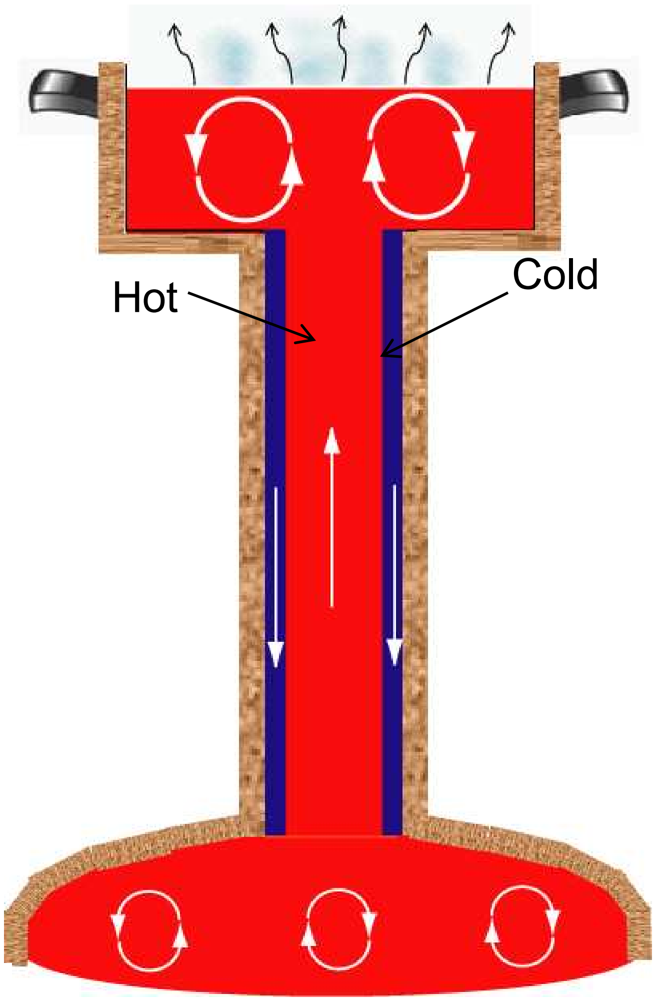
Gerst et al., 2008

Passive degassing (lake oscillation)

Courtesy from C.Oppenheimer

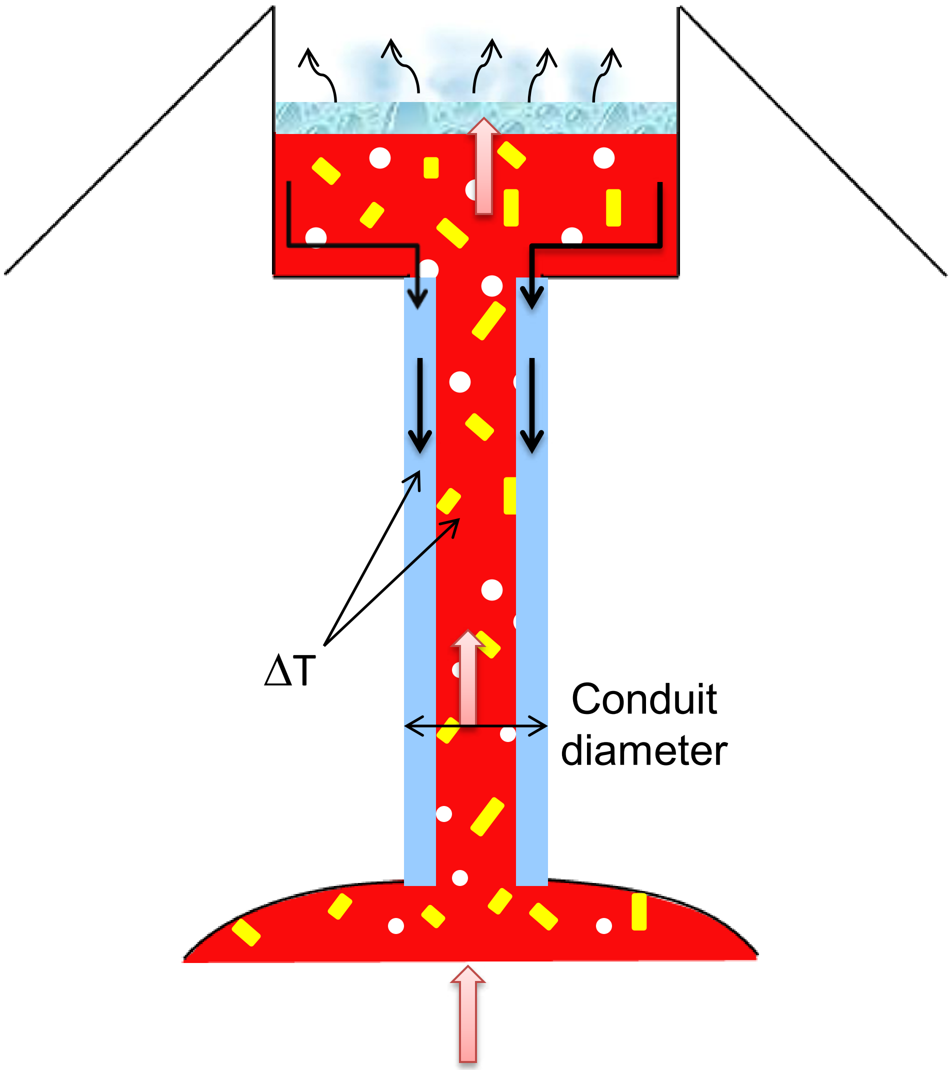
Specific objectives





Jaupart and Brandeis (1986)
Kazahaya et al. (1994)
Stevenson and Blake (1998)
Huppert and Hallworth (2007)
Molina et al. (2012)

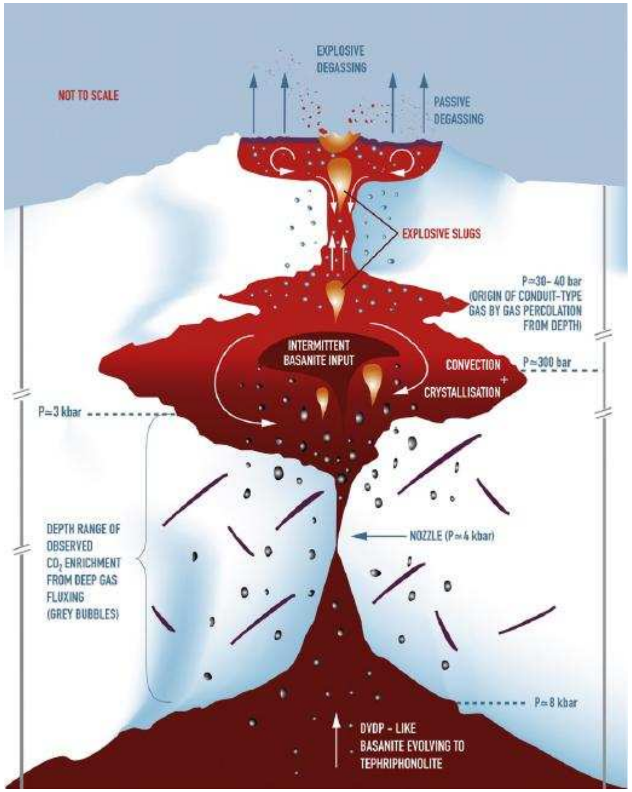
Specific objectives



1. Can we simulate the convective regime with fluid dynamics?
2. Which conduit diameter is sufficient to sustain convection in Erebus?
3. Can crystals be part of the melt?
4. Which ΔT and crystal content characterize convective currents?
5. Explain the role of crystals.
6. Explain the role of bubbles.

2. Numerical modeling

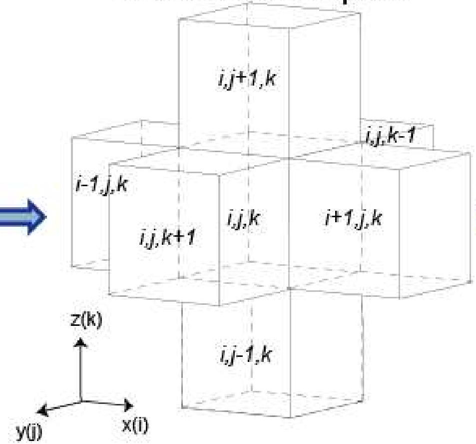
Methodology of numerical simulations



Oppenheimer et al. (2011)

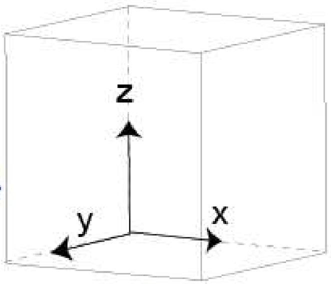
(b)

Control volume
oriented in the space



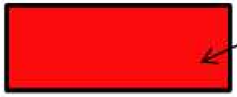
(c)

Control volume



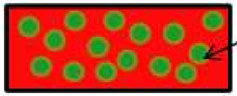
(d)

One phase



Continous

Two phases

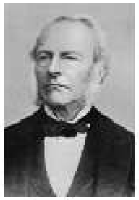


grains

(e)

Newtonian fluid

Navier



Stokes

+

non Newtonian fluid

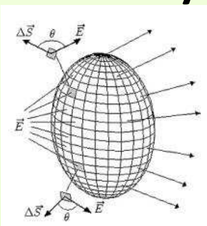
Continuity
Momentum
Energy

(f)

Equations are integrated,
linearized and solved by linear equation
solving technique
MFIX

Courtesy L. Catherine

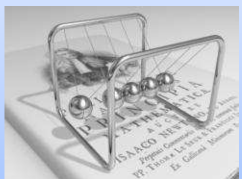
Continuity



Accumulation of mass per volume; Convective flux of mass per volume; Rate of production of n-specie

$$\frac{\partial}{\partial t}(\varepsilon_k \rho_k) + \nabla.(\varepsilon_k \rho_k \vec{v}_k) = R_{kl}$$

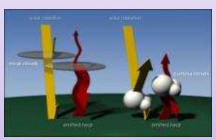
Momentum



Rate of increase of momentum per volume; Rate of momentum transferred by convection; Surface forces; Interface forces; Body forces

$$\frac{\partial}{\partial t}(\varepsilon_k \rho_k \vec{v}_k) + \nabla.(\varepsilon_k \rho_k \vec{v}_k \vec{v}_k) = \nabla.\overline{\overline{S}}_k + \overrightarrow{I_{kl}} + \overrightarrow{f_k}$$

Energy



Net rate of change of temperature per volume; Heat of conduction; Heat interphase transfer; Heat of reaction

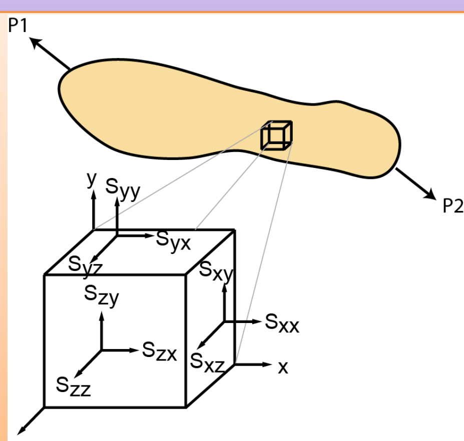
$$\varepsilon_k \rho_k C_{pk}\left(\frac{\partial T_k}{\partial t} + \vec{v}_k.\nabla T_k\right) = -\nabla \cdot \vec{q}_k + \gamma_{kl}(T_l - T_k) - \Delta H_k$$

List of symbols

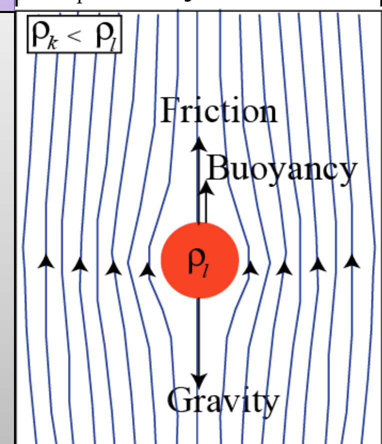
- $\varepsilon_k \rightarrow$ melt fraction
- $\rho_k \rightarrow$ density
- $P_k \rightarrow$ pressure
- $\vec{v}_k \rightarrow$ velocity
- $\mu_k \rightarrow$ viscosity
- $R_{kl} \rightarrow$ reaction rate
- $\overline{\overline{S}}_k \rightarrow$ stress tensor
- $C_{pk} \rightarrow$ heat capacity
- $T_k \rightarrow$ temperature
- $t \rightarrow$ time
- $\overline{\overline{\tau}}_k \rightarrow$ shear stress
- $d_l \rightarrow$ particle diameter
- $Re_l \rightarrow$ Reynolds

$$\overline{\overline{S}}_k = P_k \overline{\overline{I}} + \overline{\overline{\tau}}_k$$

$$\overline{\overline{\tau}}_k : f(\mu_k)$$

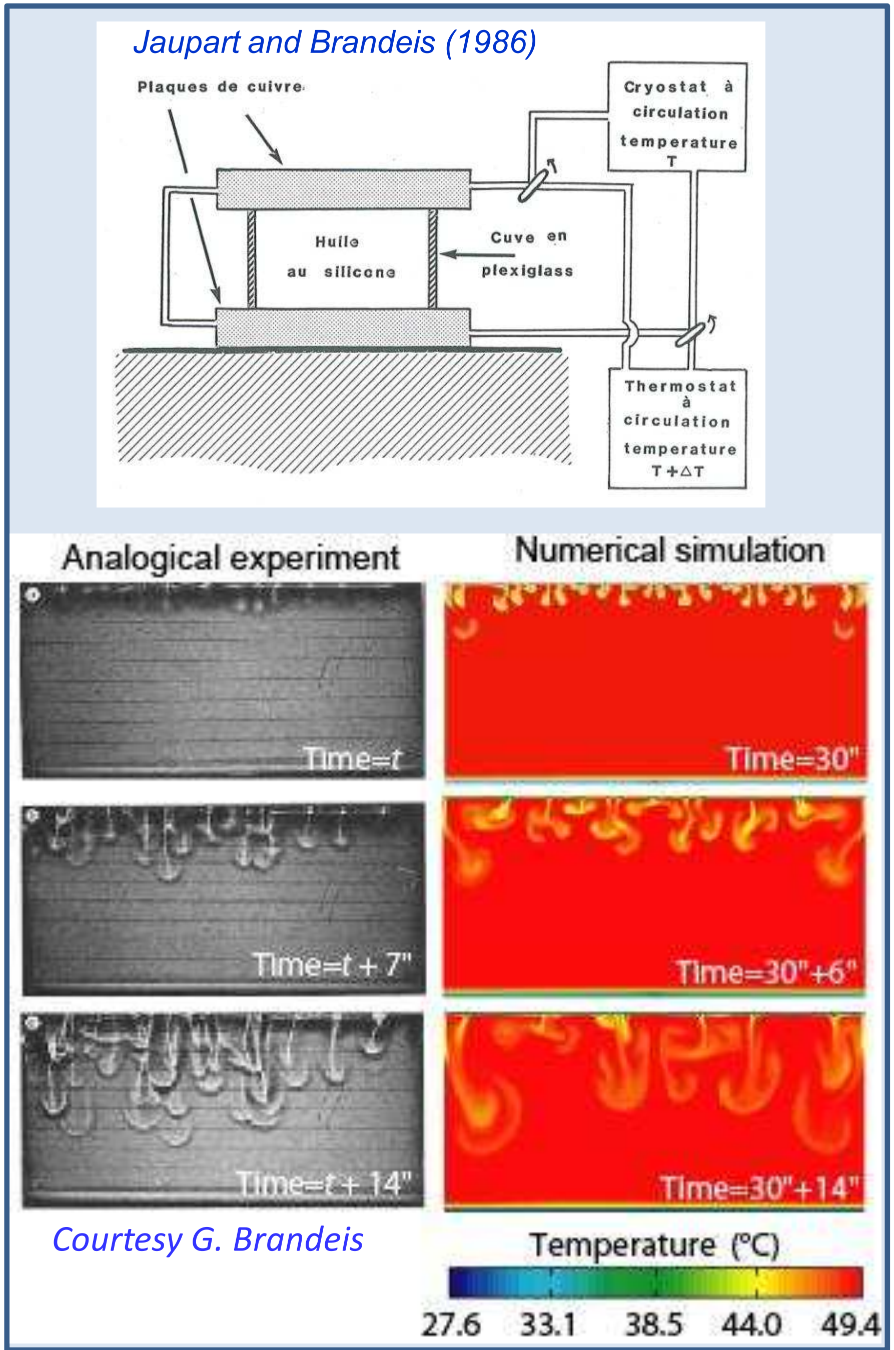


$$\overrightarrow{I_{kl}} = f\left(\varepsilon_l, \varepsilon_k, \rho_k, d_l, Re_l, \overrightarrow{v_l}, \overrightarrow{v_k}\right)$$



2. Numerical modeling

Validation (melt)



1. Continuity (Equation)

$\varepsilon_m + \varepsilon_s = 1$ — T1

Melt

$\frac{\partial}{\partial t}(\varepsilon_m \rho_m) + \nabla \cdot (\varepsilon_m \rho_m \mathbf{v}_m) = 0$ — T2

Solid

$\frac{\partial}{\partial t}(\varepsilon_s \rho_s) + \nabla \cdot (\varepsilon_s \rho_s \mathbf{v}_s) = 0$ — T3

2. Momentum

Melt

$\frac{\partial}{\partial t}(\varepsilon_m \rho_m \mathbf{v}_m) + \nabla \cdot (\varepsilon_m \rho_m \mathbf{v}_m \mathbf{v}_m) = \nabla \cdot \mathbf{S}_m + \varepsilon_m \rho_m \mathbf{g} - F_{ms}(\mathbf{v}_m - \mathbf{v}_s)$ — T4

Solid

$\frac{\partial}{\partial t}(\varepsilon_s \rho_s \mathbf{v}_s) + \nabla \cdot (\varepsilon_s \rho_s \mathbf{v}_s \mathbf{v}_s) = \nabla \cdot \mathbf{S}_s + \varepsilon_s \rho_s \mathbf{g} + F_{ms}(\mathbf{v}_m - \mathbf{v}_s) - \varepsilon_s \nabla P_m$ — T5

Stress Tensor

Melt

$\mathbf{S}_m = -P_m \mathbf{I} + \boldsymbol{\tau}_m$ — T6

Granular

$\mathbf{S}_s = -P_s^P \mathbf{I} + \boldsymbol{\tau}_s^P$ if $\varepsilon_m \le \varepsilon_m^*$ Plastic / Frictional — T7

$\mathbf{S}_s = -P_s^v \mathbf{I} + \boldsymbol{\tau}_s^v$ if $\varepsilon_m > \varepsilon_m^*$ Kinetic - Collisional — T8

Granular normal stress

$P_s^P = 10^{24}(\varepsilon_m^* - \varepsilon_m)^{10} \varepsilon_s$ — T9

$P_s^v = 2(1+e)\rho_s g_o \varepsilon_s \Theta_s$ — T10

Where $g_o = f(\varepsilon_m, \varepsilon_s)$ — T11

$\Theta_s = f(K_1, K_2, K_3, K_4, \varepsilon_s, tr(\mathbf{D}_s))$ — T12

$K_1 = f(e, g_o, \rho_s)$ — T13

$K_2 = f(e, g_o, \rho_s, d_p, \varepsilon_s, K_3)$ — T14

$K_3 = f(e, g_o, \rho_s, d_p, \varepsilon_s)$ — T15

$K_4 = f(e, g_o, \rho_s, d_p)$ — T16

Melt viscous stress

$\boldsymbol{\tau}_m = 2\varepsilon_m \mu_m \mathbf{D}_m - \frac{2}{3}\varepsilon_m \mu_m tr(\mathbf{D}_m)\mathbf{I}$ — T17

Granular viscous stress

$\boldsymbol{\tau}_s^P = \min\left[2\mu_s^P \mathring{\bar{\bar{\mathbf{D}}}}_s, 2\mu_s^{max} \mathring{\bar{\bar{\mathbf{D}}}}_s\right]$ where $\mu_s^P = \frac{P_s^P \sin\phi}{2\sqrt{I_{2D}}}$; $\mu_s^{max} = 100$ Pa s — T18

$\boldsymbol{\tau}_s^v = 2\mu_s^v \mathbf{D}_s + \lambda_s^v tr(\mathbf{D}_s)\mathbf{I}$ where $\mu_s^v = K_3 \varepsilon_s \sqrt{\Theta_s}$ — T19

$\lambda_s^v = K_2 \varepsilon_s \sqrt{\Theta_s}$ — T20

Momentum Interface Transfer Coefficient

Drag forces: $F_{ms} = f(\varepsilon_s, \varepsilon_m, \rho_m, d_p, Re_s, V_r, \mathbf{v}_s, \mathbf{v}_m)$ — T21

Terminal velocity: $V_r = f(\varepsilon_m, Re_s)$ — T22

Particle Reynolds Number: $Re_s = f(d_p, \rho_m, \mu_m, \mathbf{v}_s, \mathbf{v}_m)$ — T23

3. Energy

Melt

$\varepsilon_m \rho_m C_{pm}\left(\frac{\partial T_m}{\partial t} + \mathbf{v}_m \cdot \nabla T_m\right) = -\nabla \cdot \mathbf{q}_m + \gamma_{ms}(T_s - T_m)$ — T24

Solid

$\varepsilon_s \rho_s C_{ps}\left(\frac{\partial T_s}{\partial t} + \mathbf{v}_s \cdot \nabla T_s\right) = -\nabla \cdot \mathbf{q}_s - \gamma_{ms}(T_s - T_m)$ — T25

Heat conductivity

Melt conductivity: $\mathbf{q}_m = \varepsilon_m k_m \nabla T_m$ — T26

Granular conductivity: $\mathbf{q}_s = \varepsilon_s k_s \nabla T_s$ — T27

Heat Interface Transfer Coefficient

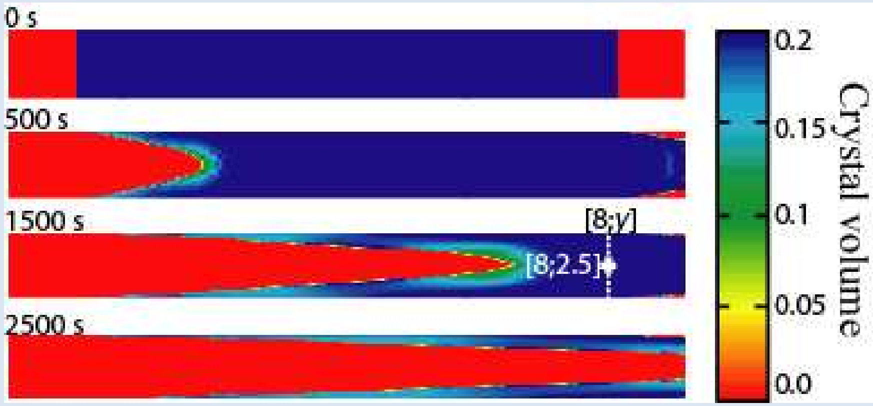
$\gamma_{ms} = f(k_m, \varepsilon_s, Nu, d_p)$ — T28

Nusselt Number: $Nu = f(\varepsilon_s, \varepsilon_m, Re_s, Pr)$ — T29

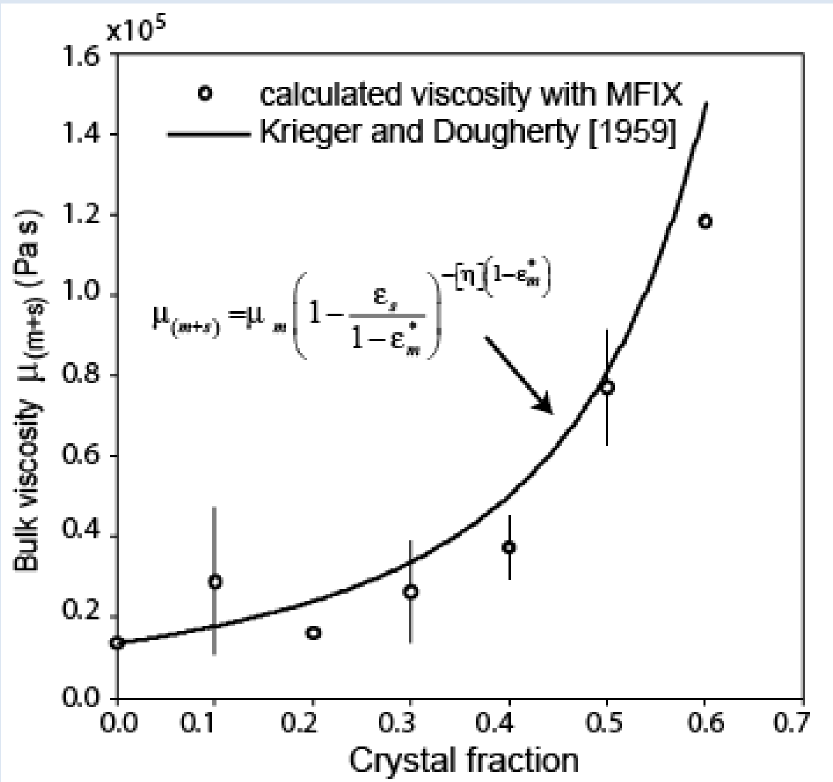
Prandtl Number: $Pr = f(C_{pm}, \mu_m, k_m)$ — T30

2. Numerical modeling

Validation (melt plus crystals)



$$-\frac{\partial P_{(m+s)}}{\partial x} + \mu_{(m+s)} \frac{\partial^2 U_{(m+s)}}{\partial y^2} = 0$$



1. Continuity	Equation
$\varepsilon_m + \varepsilon_s = 1$	T1
Melt $\frac{\partial}{\partial t}(\varepsilon_m \rho_m) + \nabla \cdot (\varepsilon_m \rho_m \mathbf{v}_m) = 0$	T2
Solid $\frac{\partial}{\partial t}(\varepsilon_s \rho_s) + \nabla \cdot (\varepsilon_s \rho_s \mathbf{v}_s) = 0$	T3
2. Momentum	
Melt $\frac{\partial}{\partial t}(\varepsilon_m \rho_m \mathbf{v}_m) + \nabla \cdot (\varepsilon_m \rho_m \mathbf{v}_m \mathbf{v}_m) = \nabla \cdot \mathbf{S}_m + \varepsilon_m \rho_m \mathbf{g} - F_{ms}(\mathbf{v}_m - \mathbf{v}_s)$	T4
Solid $\frac{\partial}{\partial t}(\varepsilon_s \rho_s \mathbf{v}_s) + \nabla \cdot (\varepsilon_s \rho_s \mathbf{v}_s \mathbf{v}_s) = \nabla \cdot \mathbf{S}_s + \varepsilon_s \rho_s \mathbf{g} + F_{ms}(\mathbf{v}_m - \mathbf{v}_s) - \varepsilon_s \nabla P_m$	T5
Stress Tensor	
Melt: $\mathbf{S}_m = -P_m \mathbf{I} + \boldsymbol{\tau}_m$ T6	
Granular: $\mathbf{S}_s = -P_s^P \mathbf{I} + \boldsymbol{\tau}_s^P$ if $\varepsilon_m \le \varepsilon_m^*$ Plastic / Frictional	T7
$\mathbf{S}_s = -P_s^v \mathbf{I} + \boldsymbol{\tau}_s^v$ if $\varepsilon_m > \varepsilon_m^*$ Kinetic - Collisional	T8
Granular normal stress	
$P_s^P = 10^{24}(\varepsilon_m^* - \varepsilon_m)^{10} \varepsilon_s$ T9 ; $K_1 = f(e, g_o, \rho_s)$ T13	
$P_s^v = 2(1+e)\rho_s g_o \varepsilon_s \Theta_s$ T10 ; $K_2 = f(e, g_o, \rho_s, d_p, \varepsilon_s, K_3)$ T14	
Where $g_o = f(\varepsilon_m, \varepsilon_s)$ T11 ; $K_3 = f(e, g_o, \rho_s, d_p, \varepsilon_s)$ T15	
$\Theta_s = f(K_1, K_2, K_3, K_4, \varepsilon_s, tr(\mathbf{D}_s))$ T12 ; $K_4 = f(e, g_o, \rho_s, d_p)$ T16	
Melt viscous stress: $\boldsymbol{\tau}_m = 2\varepsilon_m \mu_m \mathbf{D}_m - \frac{2}{3}\varepsilon_m \mu_m tr(\mathbf{D}_m)\mathbf{I}$ T17	
Granular viscous stress: $\boldsymbol{\tau}_s^P = \min\left[2\mu_s^P \mathring{\bar{\bar{\mathbf{D}}}}_s, 2\mu_s^{max} \mathring{\bar{\bar{\mathbf{D}}}}_s\right]$ where $\mu_s^P = \frac{P_s^P \sin\phi}{2\sqrt{I_{2D}}}; \mu_s^{max} = 100$ Pa s	T18
$\boldsymbol{\tau}_s^v = 2\mu_s^v \mathbf{D}_s + \lambda_s^v tr(\mathbf{D}_s)\mathbf{I}$ where $\mu_s^v = K_3 \varepsilon_s \sqrt{\Theta_s}$	T19
$\lambda_s^v = K_2 \varepsilon_s \sqrt{\Theta_s}$	T20
Momentum Interface Transfer Coefficient	
Drag forces: $F_{ms} = f(\varepsilon_s, \varepsilon_m, \rho_m, d_p, Re_s, V_r, \mathbf{v}_s, \mathbf{v}_m)$ T21 ; Terminal velocity: $V_r = f(\varepsilon_m, Re_s)$ T22 ; Particle Reynolds Number: $Re_s = f(d_p, \rho_m, \mu_m, \mathbf{v}_s, \mathbf{v}_m)$	T23
3. Energy	
Melt $\varepsilon_m \rho_m C_{pm}\left(\frac{\partial T_m}{\partial t} + \mathbf{v}_m \cdot \nabla T_m\right) = -\nabla \cdot \mathbf{q}_m + \gamma_{ms}(T_s - T_m)$	T24
Solid $\varepsilon_s \rho_s C_{ps}\left(\frac{\partial T_s}{\partial t} + \mathbf{v}_s \cdot \nabla T_s\right) = -\nabla \cdot \mathbf{q}_s - \gamma_{ms}(T_s - T_m)$	T25
Heat conductivity	
Melt conductivity: $\mathbf{q}_m = \varepsilon_m k_m \nabla T_m$ T26 ; Granular conductivity: $\mathbf{q}_s = \varepsilon_s k_s \nabla T_s$	T27
Heat Interface Transfer Coefficient	
$\gamma_{ms} = f(k_m, \varepsilon_s, Nu, d_p)$ T28 ; Nusselt Number: $Nu = f(\varepsilon_s, \varepsilon_m, Re_s, Pr)$ T29 ; Prandtl Number: $Pr = f(C_{pm}, \mu_m, k_m)$	T30

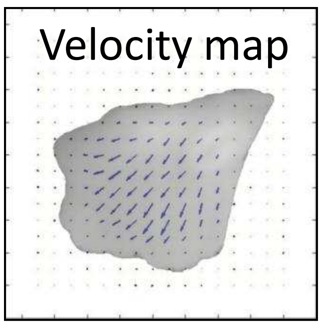
2. Numerical modeling: role of crystals — Data

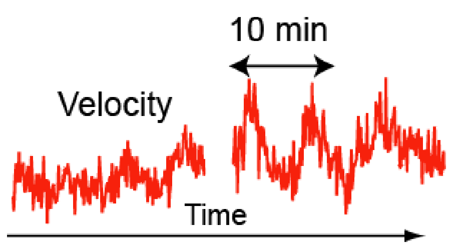
- Crystals (e.g., *Dunbar et al. 1994*; *Kelly et al. 2008*):

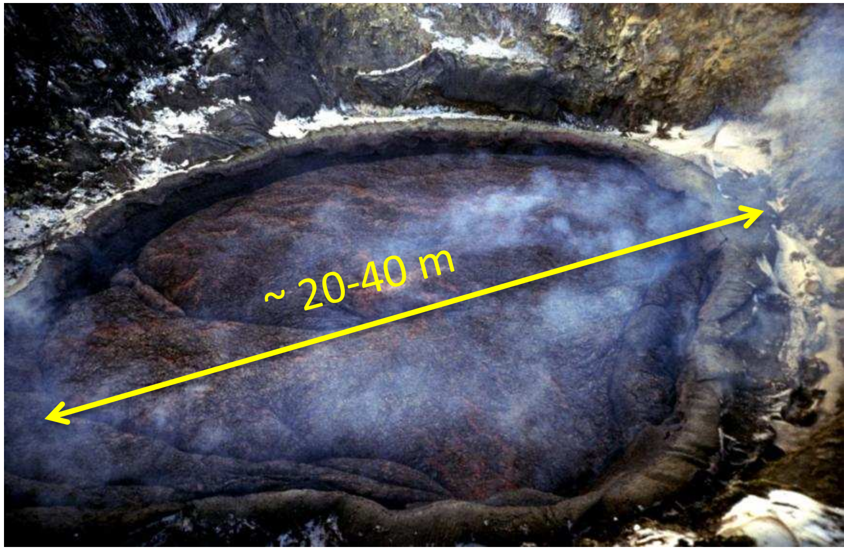

  *Megacrysts (cm-size)
  *30 vol.% anorthoclase (no experimental data).
  *Old crystals (hundreds of years)
  *No cristallization
  *No magma intrusion

- *Kyle, 1977* crystal circulation (constant magma composition).

- Surface velocities of ~0.1 m/s with a mean period of 10 min (*Oppenheimer et al. 2009*).



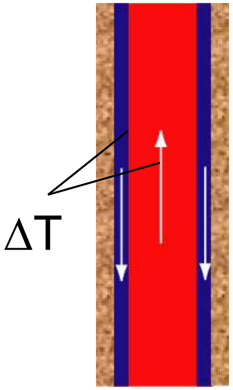




- Magma system dimensions: *McClelland et al. 1989* -> lake diameter 20 m; *Dibble et al, 2008* -> conduit 10-20(?) m; *Harris et al,1999* -> lake diameter changes in days *Oppenheimer et al. 2009* -> since 2001 lake diameter 20-40 m.

Crystals (e.g., *Dunbar et al. 1994*; *Kelly et al. 2008*):

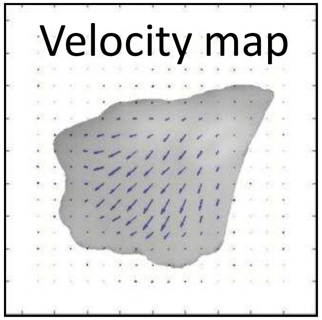
*Megacrysts (cm-size)
*30 vol.% anorthoclase (no experimental data).
*Old crystals (thousand of years)
*No cristallization
*No magma intrusion

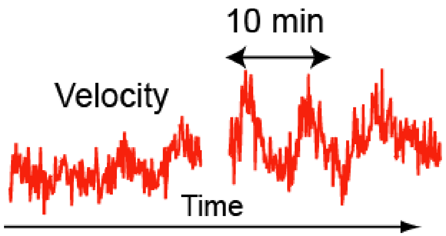


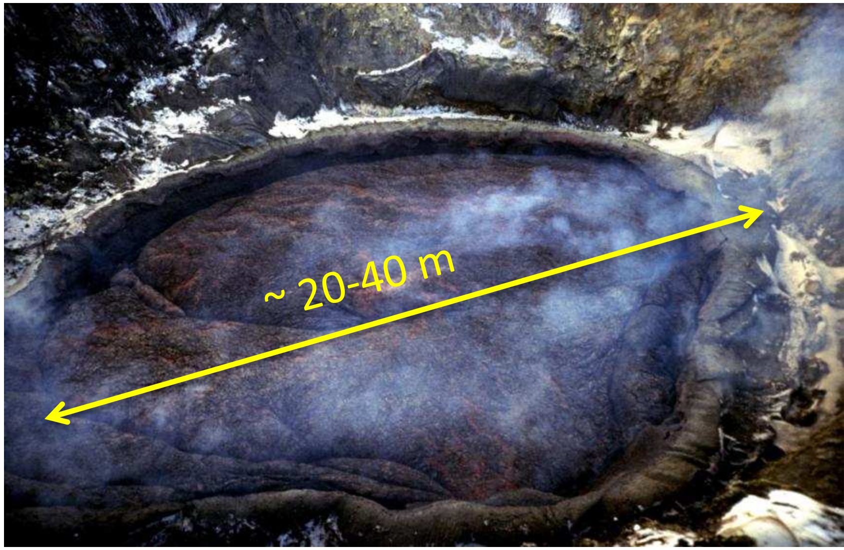
Kyle, 1977 crystal circulation (constant magma composition.

Surface velocities of ~0.1 m/s with a mean period of 10 min (*Oppenheimer et al. 2009*).

ΔT=65°C (*Sweeney et al, 2008*)
ΔT<120°C (*Calkins et al, 2008*).



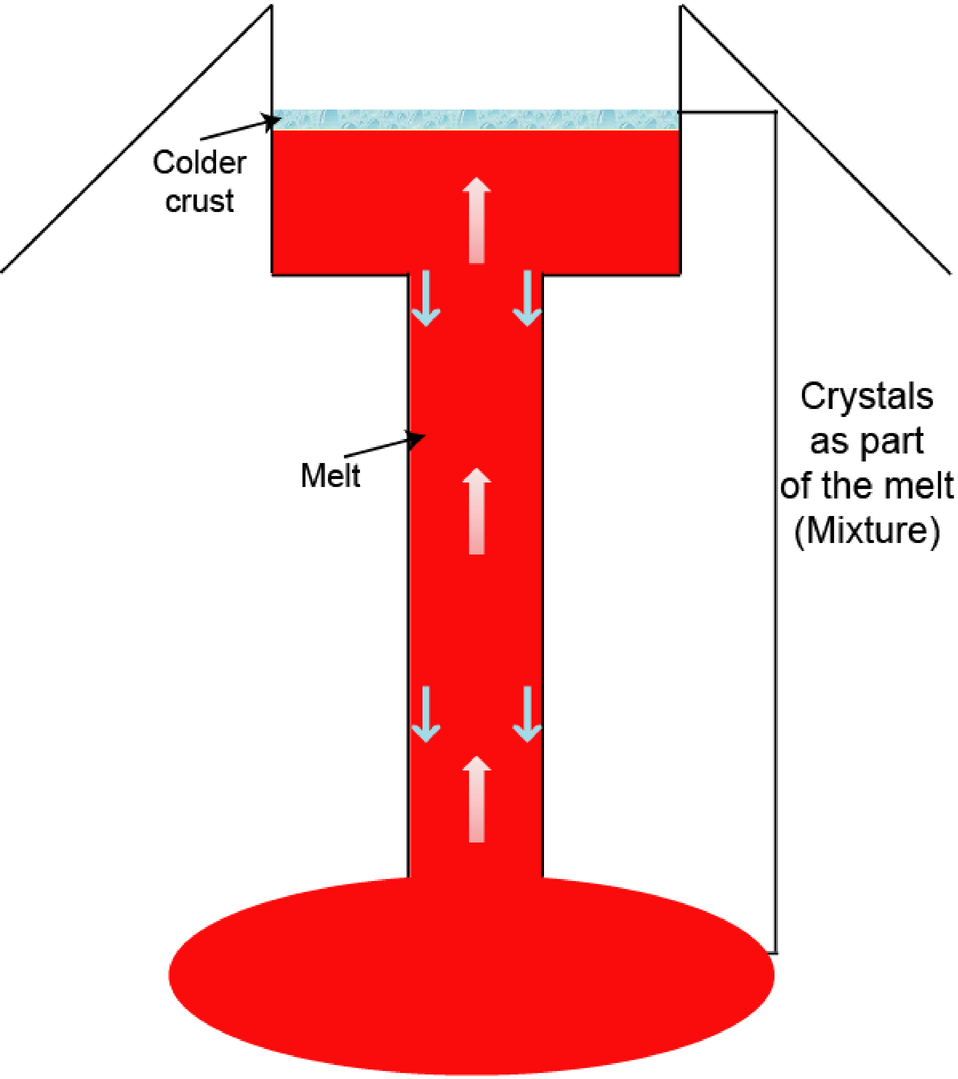


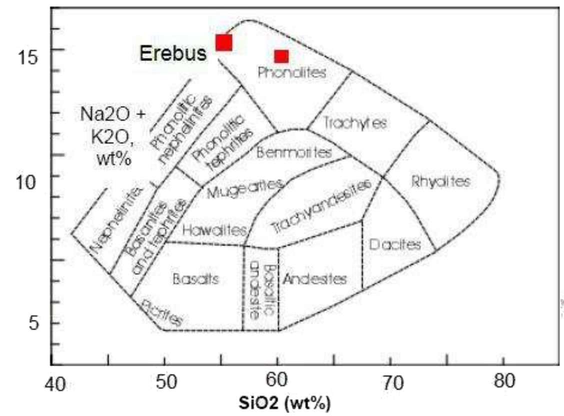


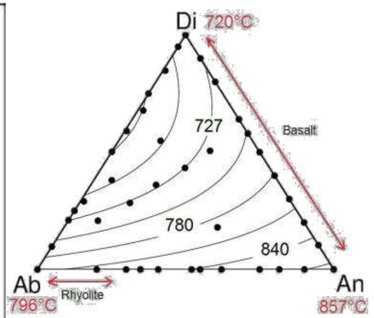
Magma system dimensions: *McClelland et al. 1989* -> lake diameter 20 m; *Dibble et al, 2008* -> conduit 10-20(?) m; *Harris et al,1999* -> lake diameter changes in days *Oppenheimer et al. 2009* -> since 2001 lake diameter 30-35 m.

Are 30% of crystals recirculating?
Can we reproduce the convective rate?
Can we reproduce the ΔT?

Closed system







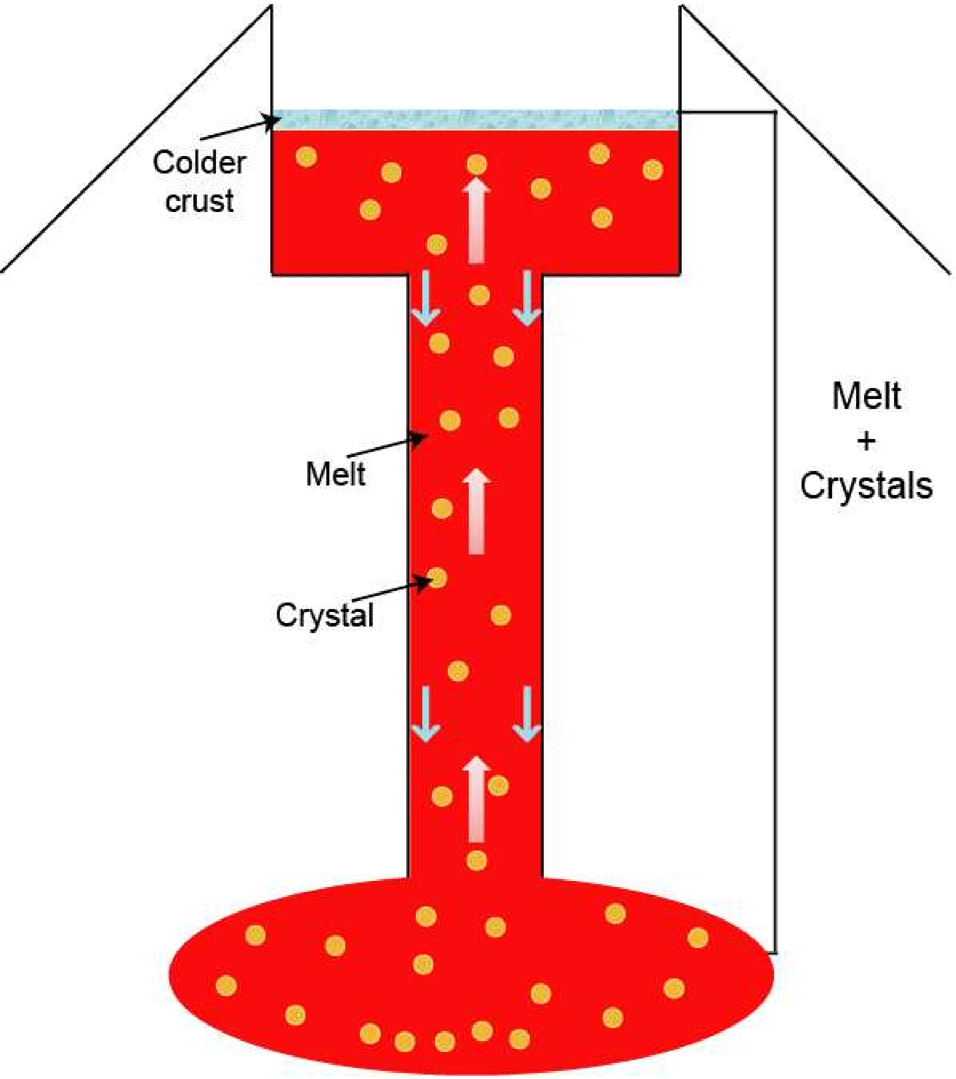
- •Constant bulk composition (*Kelly et al, 2008*).
- •Constant thermodynamical properties (e.g., heat capacity, conductivity, initial density) (*Mastin and Ghiorso, 2000*)
- •Bulk viscosity as function of T (*Giordano et al, 2008*) ->30% crystals (*Krieger and Dougherty, 1959*).

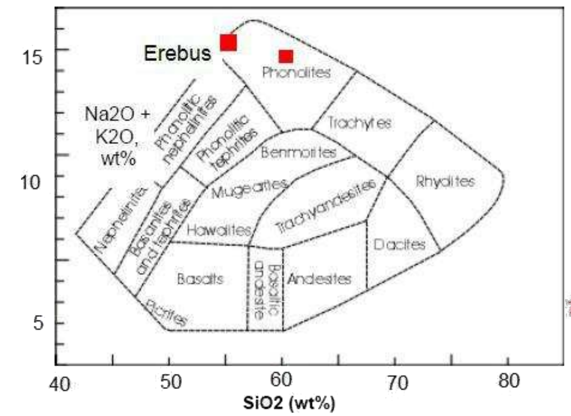
Glass transition of 700°C (*Russel and Giordano, 2005; Molina et al, 2012*)

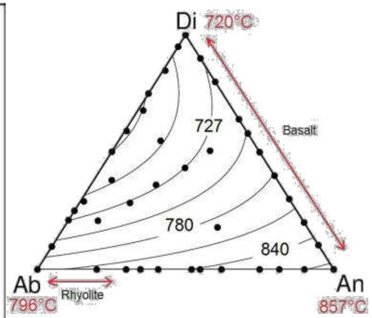
- Continuity (1 equation)
- Momentum (1 equation)
- Energy (1 equation)

→ Liquid

Closed system

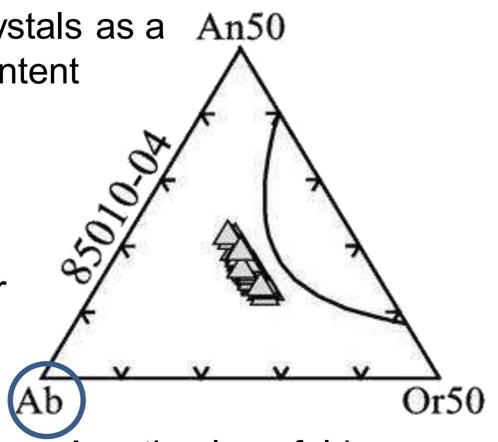






Glass transition of 700°C (*Russel and Giordano, 2005; Molina et al, 2012*)

- Constant bulk composition (*Kelly et al, 2008*).
- Constant thermodynamical properties (e.g., heat capacity, conductivity, initial density) (*Mastin and Ghiorso, 2000*)
- Bulk viscosity for melt as a function of T (*Giordano et al, 2008*).
- Bulk viscosity for crystals as a function of crystal content (*Schaeffer, 1987*)
  - Sphere
  - Rigid
  - Constant diameter
  - No crystallization

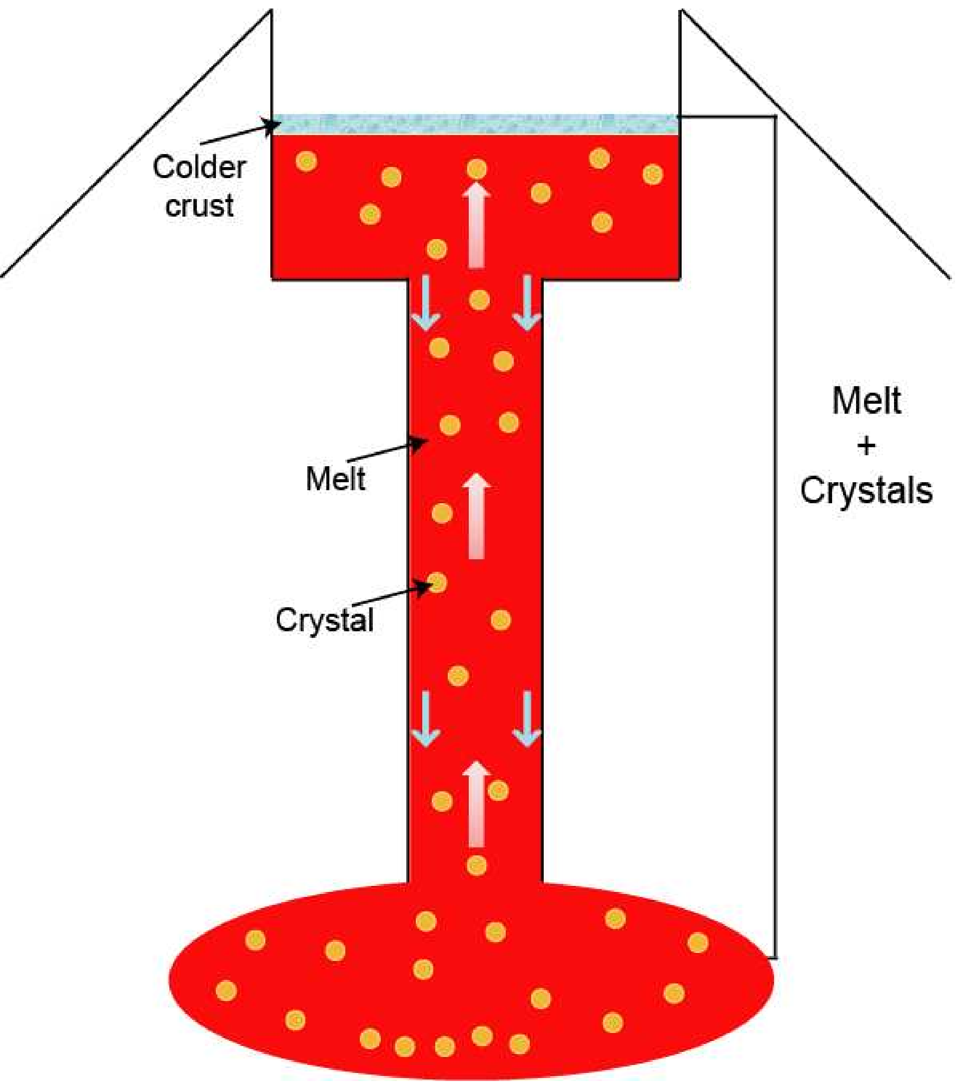


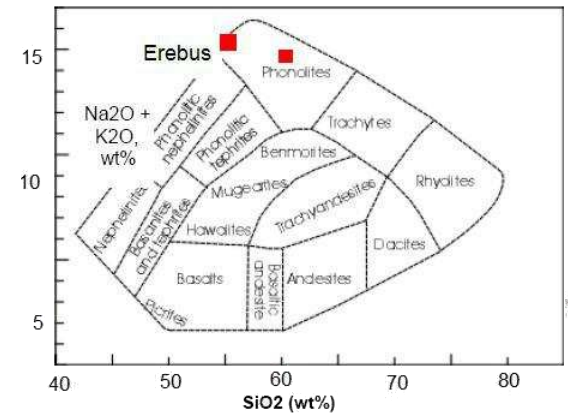
Anorthoclase feldspar composition *(Kelly et al, 2008)*

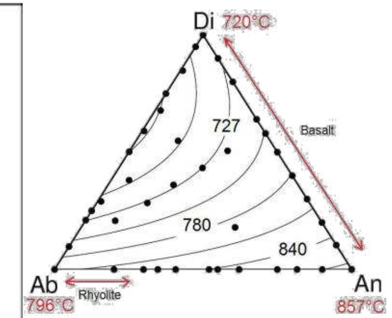
2. Numerical modeling: role of crystals

Hypotheses

Closed system







Glass transition of 700°C (*Russel and Giordano, 2005; Molina et al, 2012*)

- Constant bulk composition (*Kelly et al, 2008*).
- Constant thermodynamical properties (e.g., heat capacity, conductivity, initial density) (*Mastin and Ghiorso, 2000*)
- Bulk viscosity for melt as a function of T (*Giordano et al, 2008*).
- Bulk viscosity for crystals as a function of crystal content (*Schaeffer, 1987*)

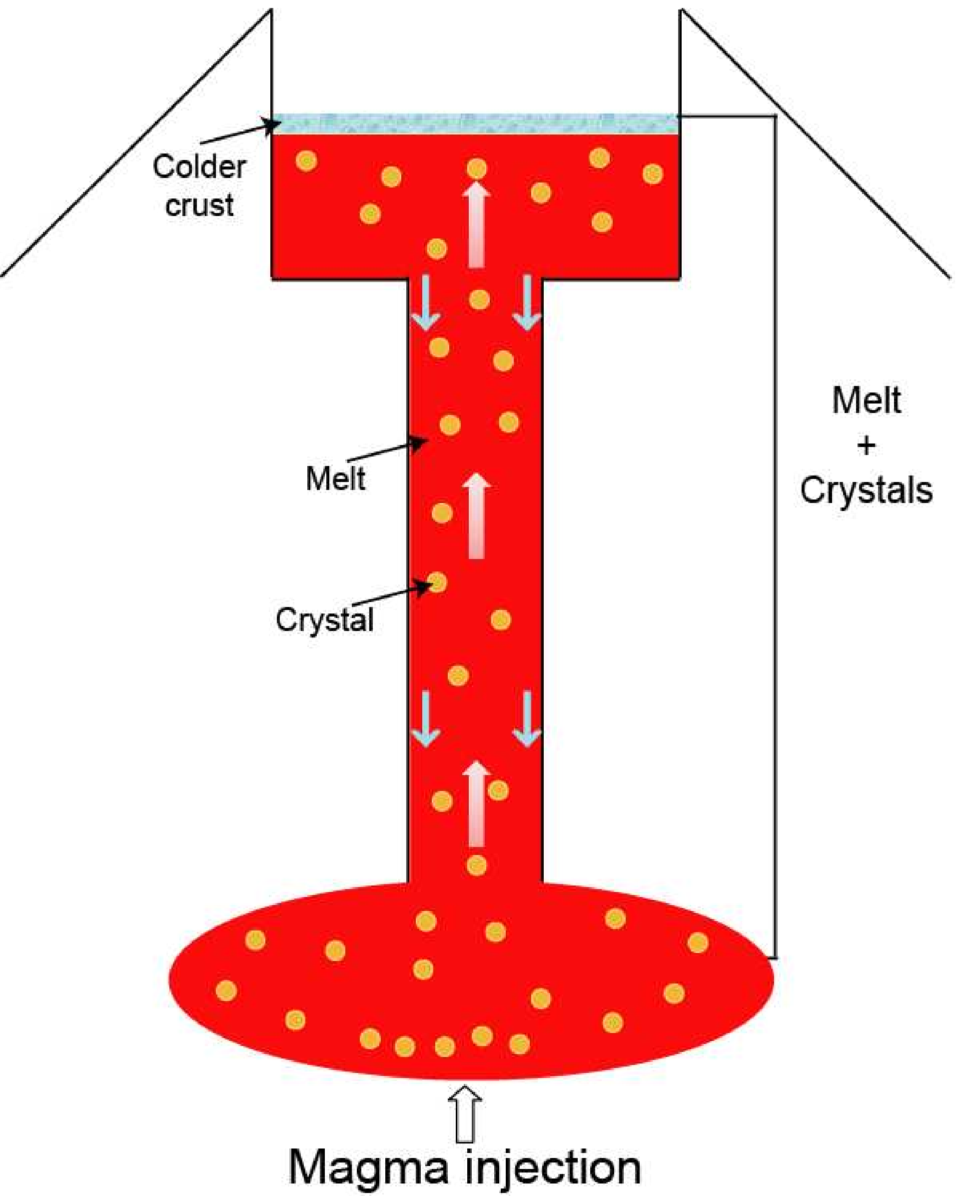
- Continuity (1 equation)
- Momentum (1 equation)
- Energy (1 equation)

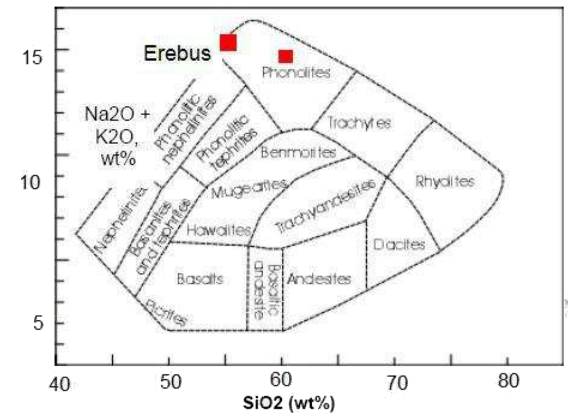
→ Liquid

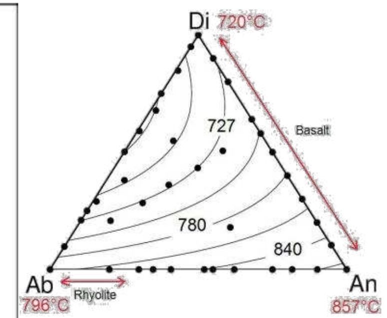
- Continuity (1 equation)
- Momentum (1 equation)
- Energy (1 equation)

→ solid

Open system







Glass transition of 700°C (*Russel and Giordano, 2005; Molina et al, 2012*)

- Constant bulk composition (*Kelly et al, 2008*).
- Constant thermodynamical properties (e.g., heat capacity, conductivity, initial density) (*Mastin and Ghiorso, 2000*)
- Bulk viscosity for melt as a function of T (*Giordano et al, 2008*).
- Bulk viscosity for crystals as a function of crystal content (*Schaeffer, 1987*)

- Continuity (1 equation)
- Momentum (1 equation)
- Energy (1 equation)

→ Liquid

- Continuity (1 equation)
- Momentum (1 equation)
- Energy (1 equation)

→ solid

2. Numerical modeling: role of crystals

Crystals as part of the melt (mixture)

One phase

Continuity (fluid)

$$\rho_{(m+s)} = \underbrace{\varepsilon_s \rho_s}_{\text{constant}} + \varepsilon_m \overbrace{\left\{\rho_o \left[1-\alpha\left(T_m - T_o\right)\right]\right\}}^{\rho_m}$$

Momentum (fluid)

$$\overline{\overline{S}}_m$$

$$\mu_{(m+s)} = \mu_m \left(1 - \frac{\varepsilon_s}{1-\varepsilon_m^*}\right)^{-[\eta]\left(1-\varepsilon_m^*\right)}$$

Krieger and Dougherty (1959)

$$\mu_m = 10\exp\left(A + \frac{B}{T_m - C}\right)$$

Giordano et al. (2008)

Symbols

ρ_m Melt density

$\rho_{(m+s)}$ Mixture density

ρ_o Initial density

α Thermal expansion

$1-\varepsilon_m^*$ MPF

$\varepsilon_s, \varepsilon_m$ Crystal/melt vol.

η Einstein coefficient

μ_m Melt viscosity

μ_m Mixture viscosity

A constant

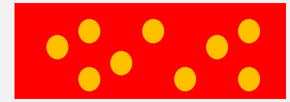
B, C f:(oxides,H_2O)
Kelly et al. (2008)
Seaman et al. (2006)

T_m, T_o Magma/initial temperature

Bulk density and viscosity definitions

Crystals as a separated phase

Two phases



Continuity (fluid+solid)

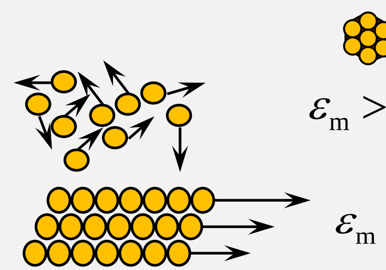
$$\rho \begin{cases} \rho_m = \varepsilon_m \left\{\rho_o \left[1-\alpha\left(T_m - T_o\right)\right]\right\} \\ \rho_s \varepsilon_s \rightarrow \text{calculated}\left(\rho_s : \text{cste}\right) \end{cases}$$

Momentum (fluid+solid)

$$\overline{\overline{S}}_m, \overline{\overline{S}}_s$$

$$\mu_m$$

$$\mu_s$$



Schaeffer (1987)

e Restitution coefficient

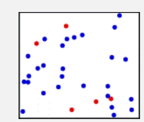
g_o Radial distribution function

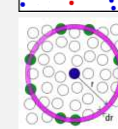
d_p Crystal diameter

Θ_s Granular temperature

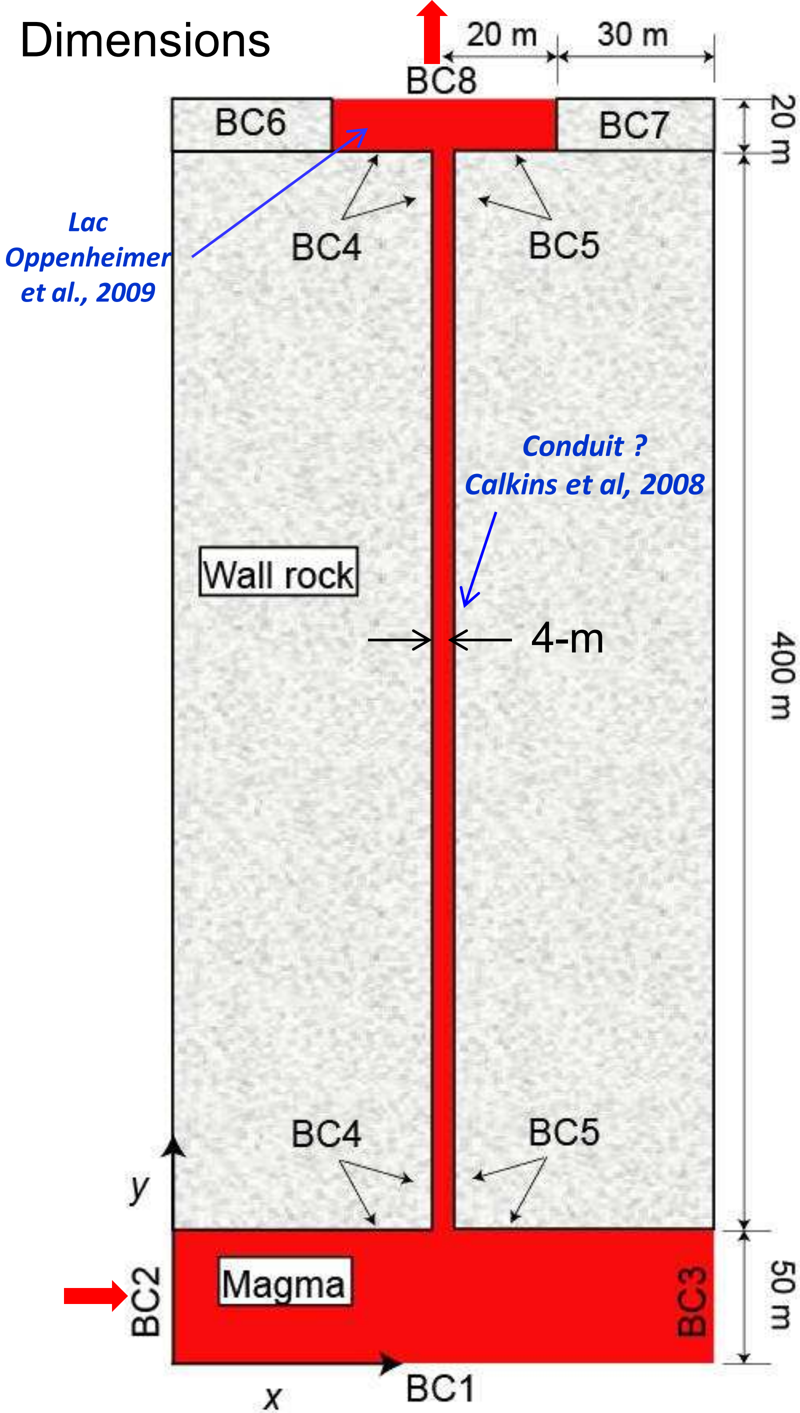
φ Friction angle

μ_s^{max} Maximum viscosity (100 Pa s)

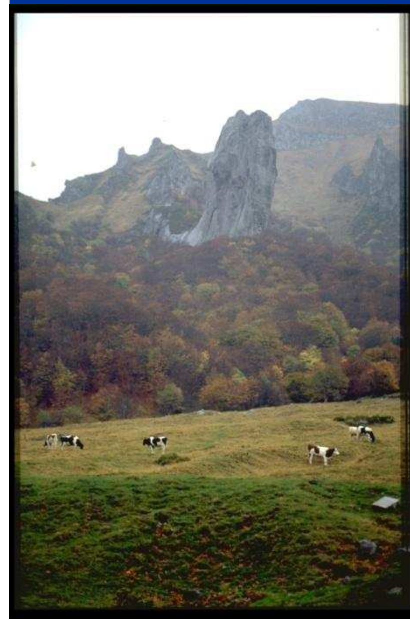




Dimensions

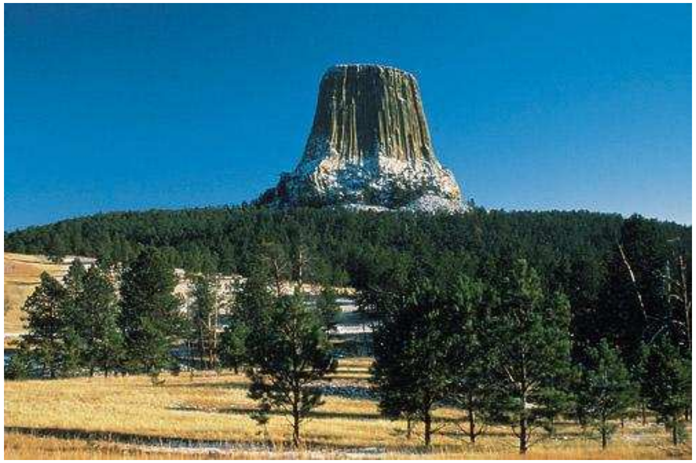


Hypotheses



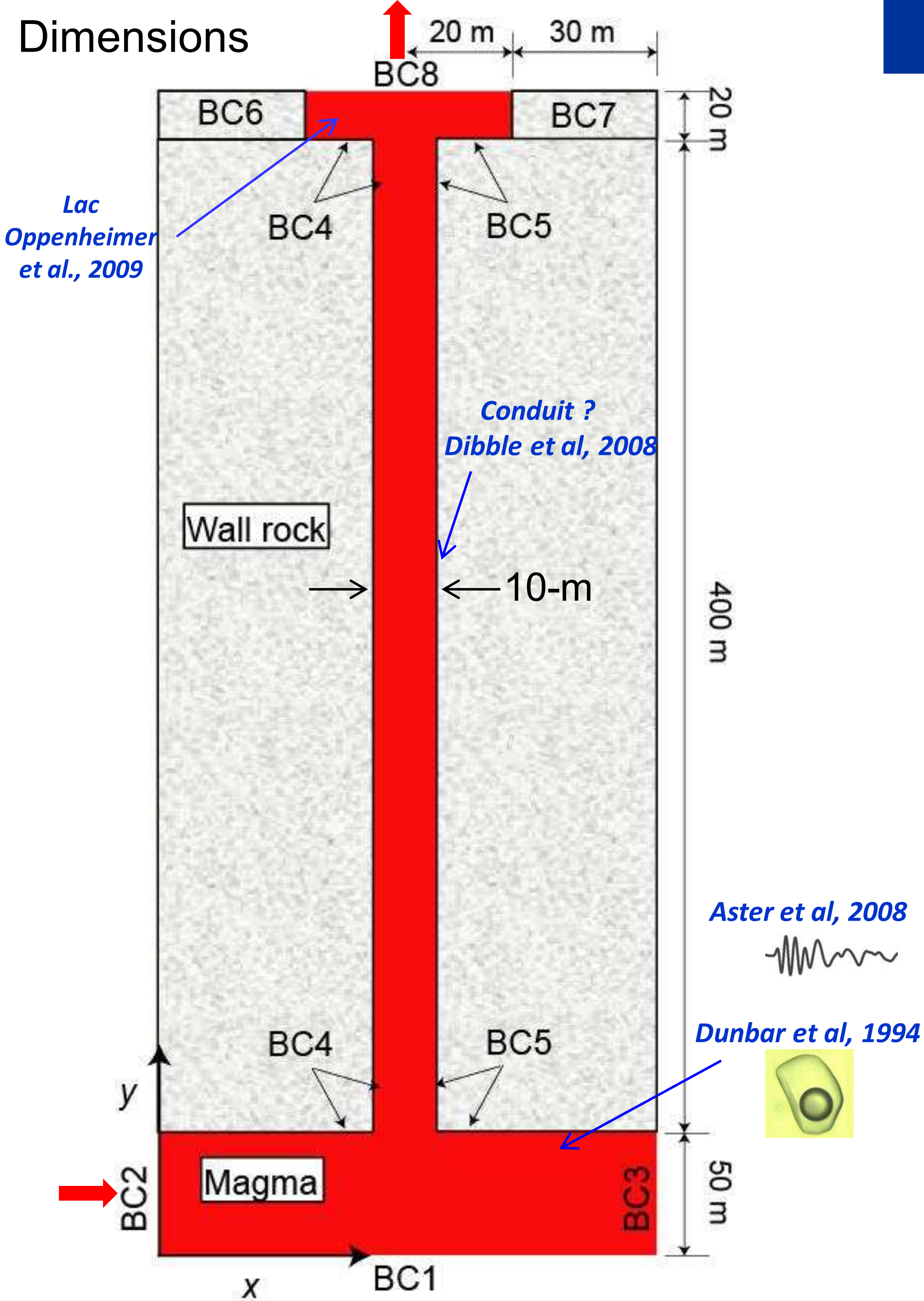
Old conduit (Massif Central, France) Courtesy S. Vergniolle

volcanic pipe (Colorado, USA)



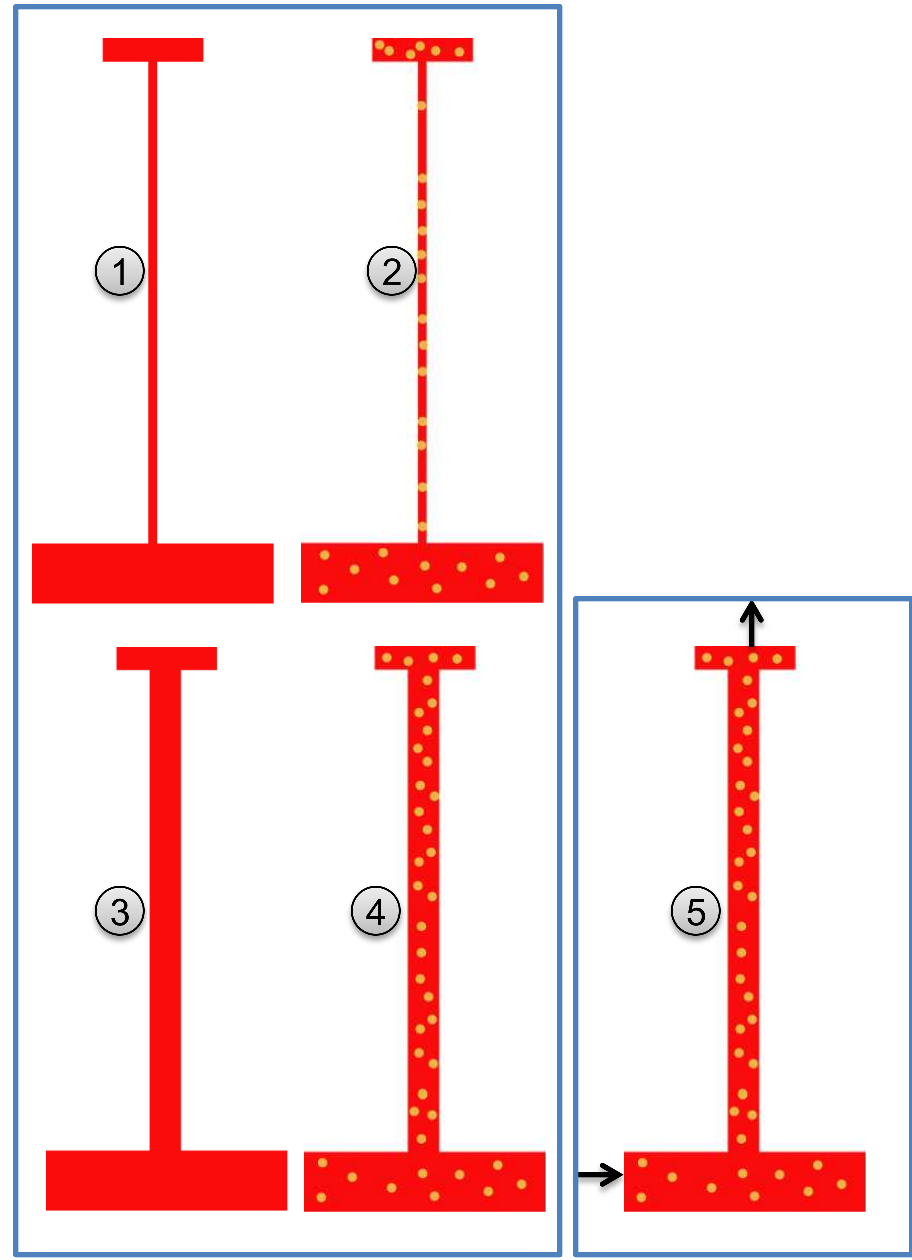
Old conduit (Wyoming,USA)

Dimensions



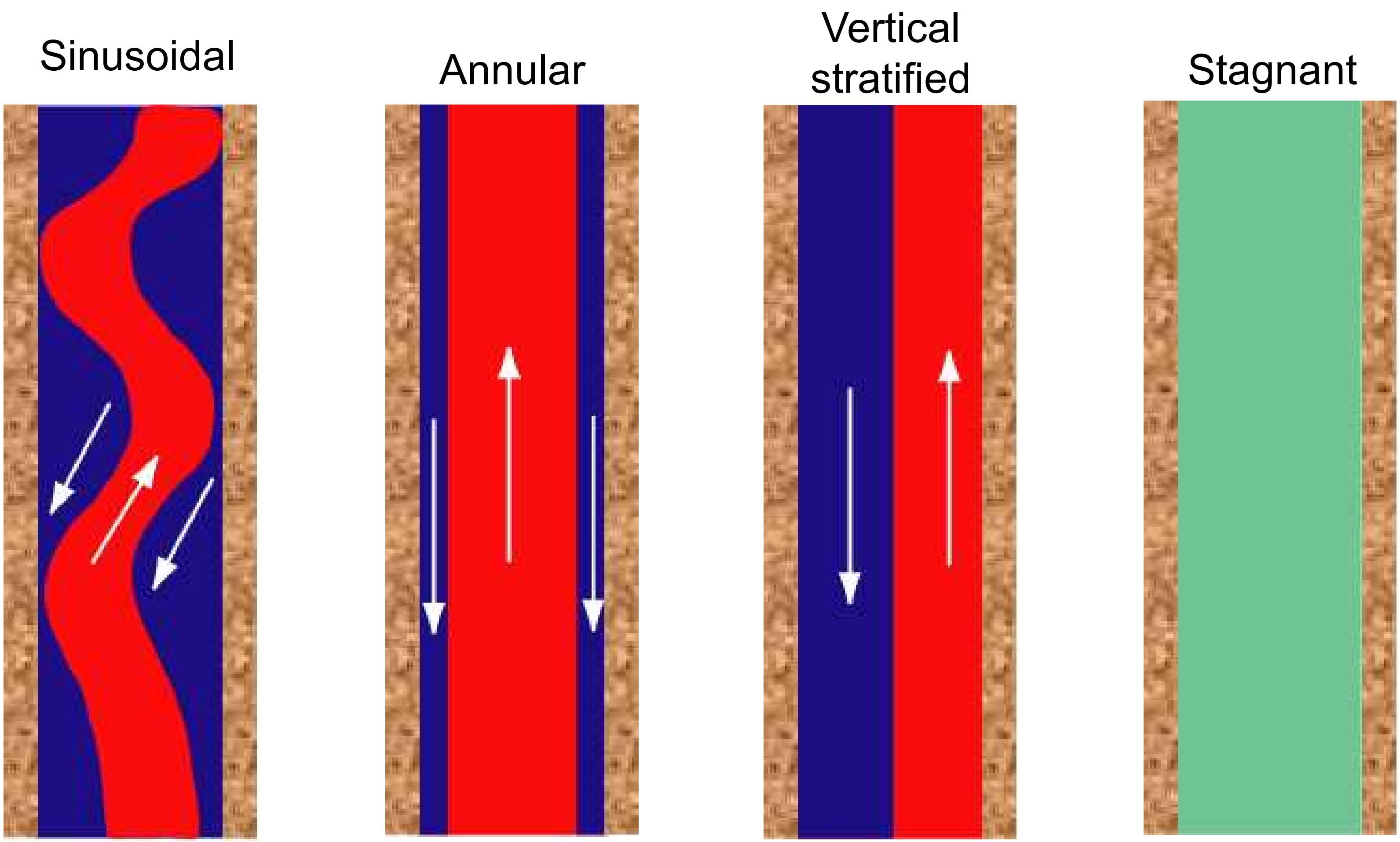
Hypotheses

Number of simulations



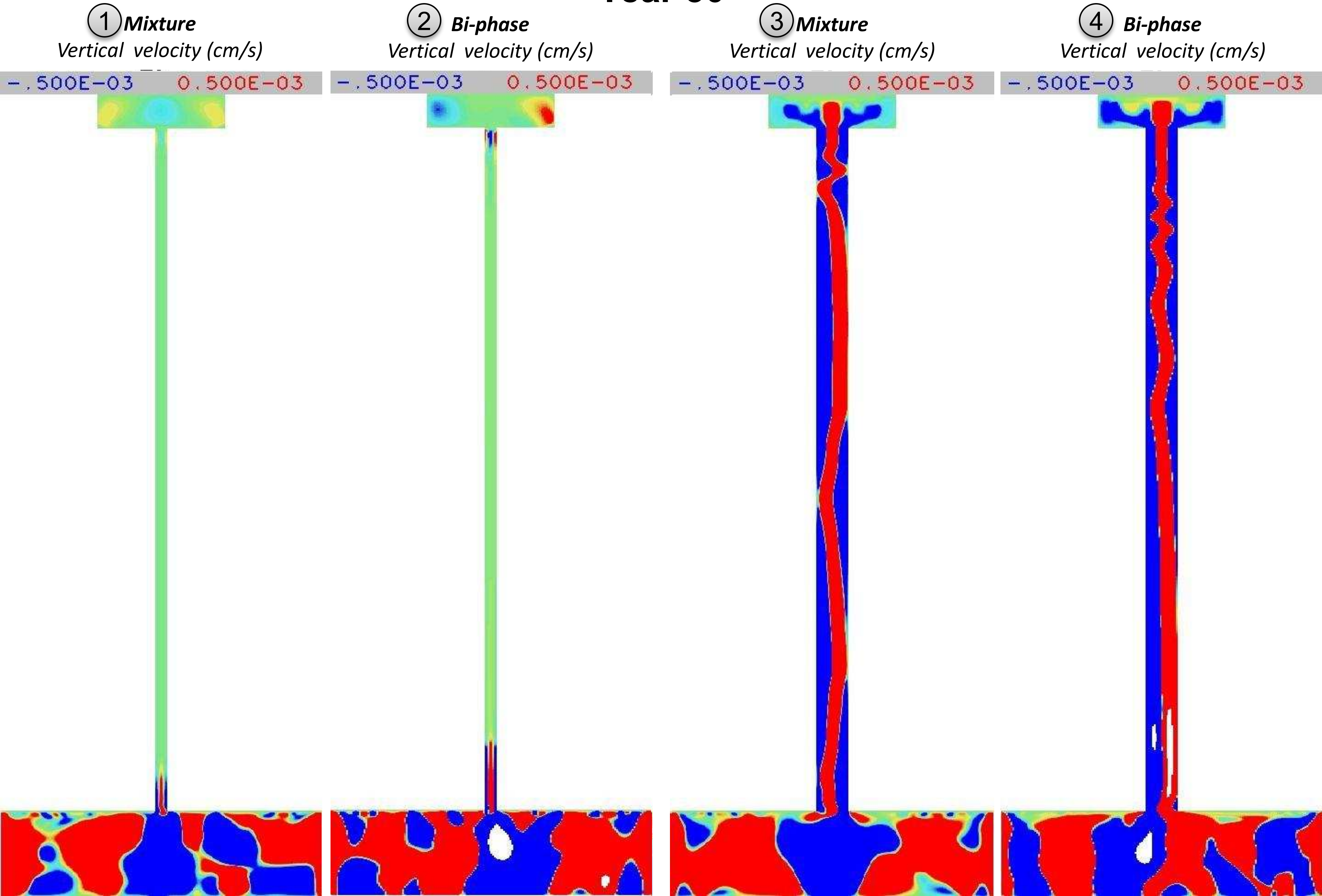
2. Numerical modeling: role of crystals

Convection patterns (velocity currents)



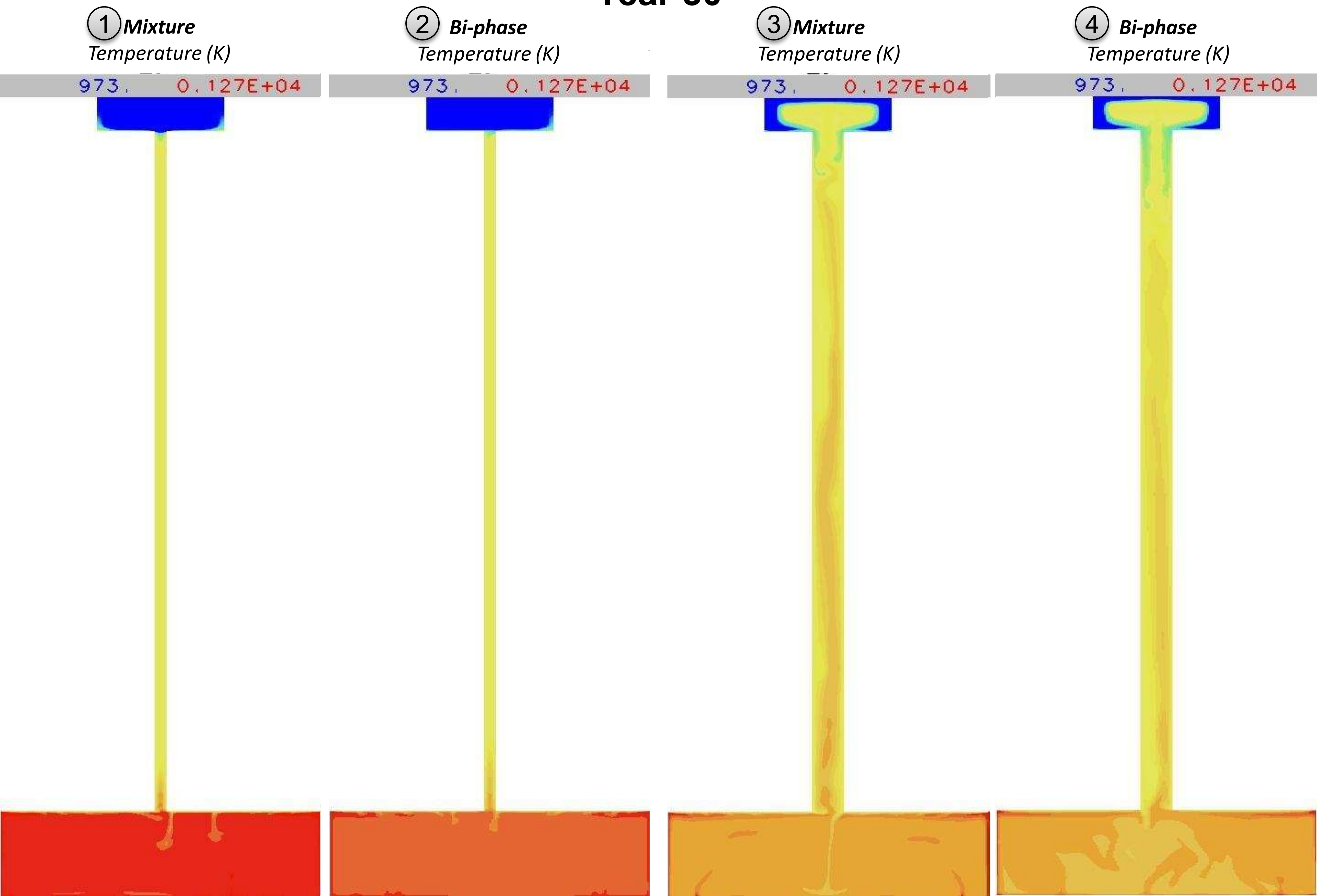
Conduit diameter -closed system-

Year 30

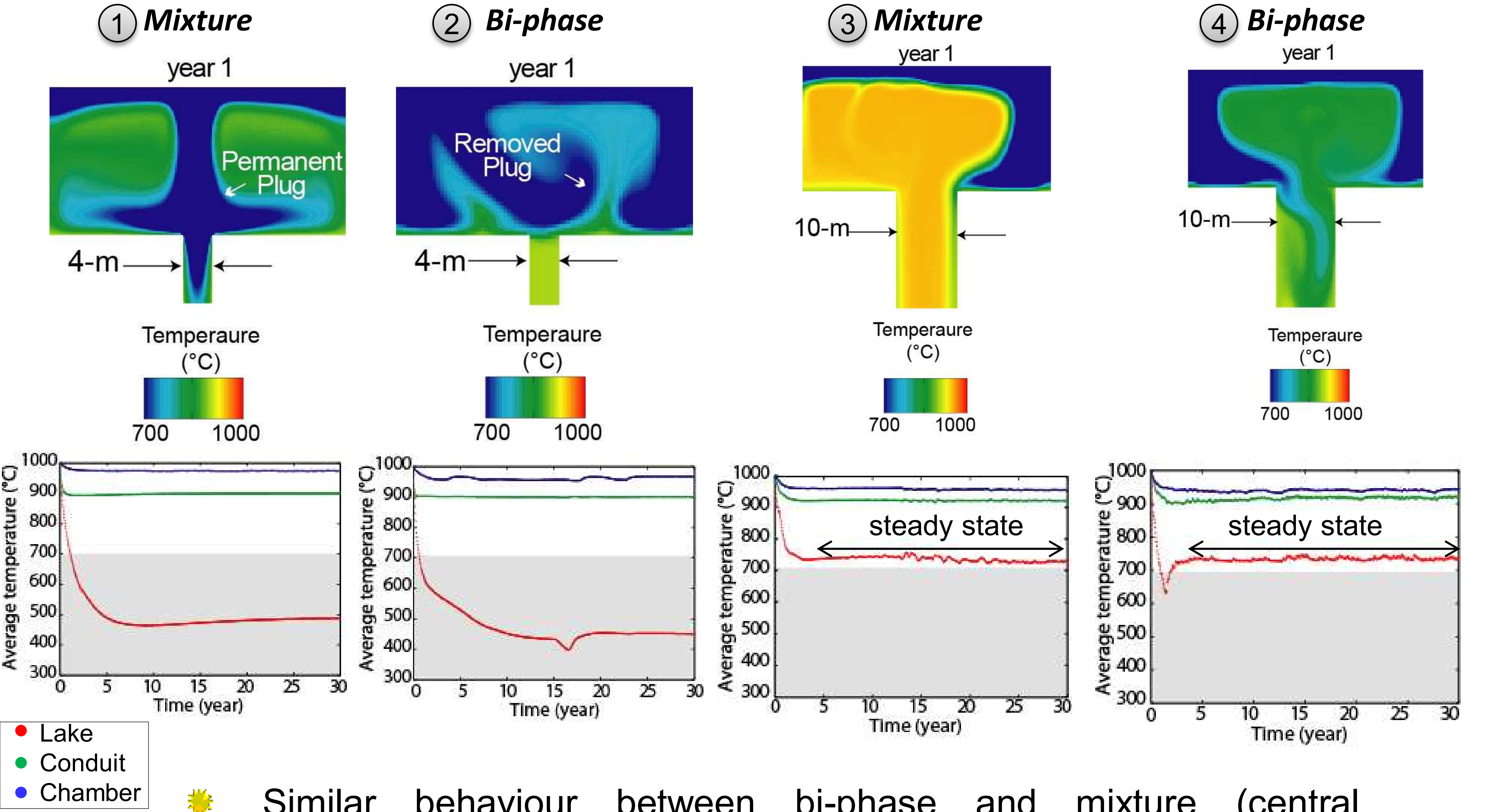


Conduit diameter -closed system-

Year 30



2. Numerical modeling: role of crystals — Conduit diameter -closed system-

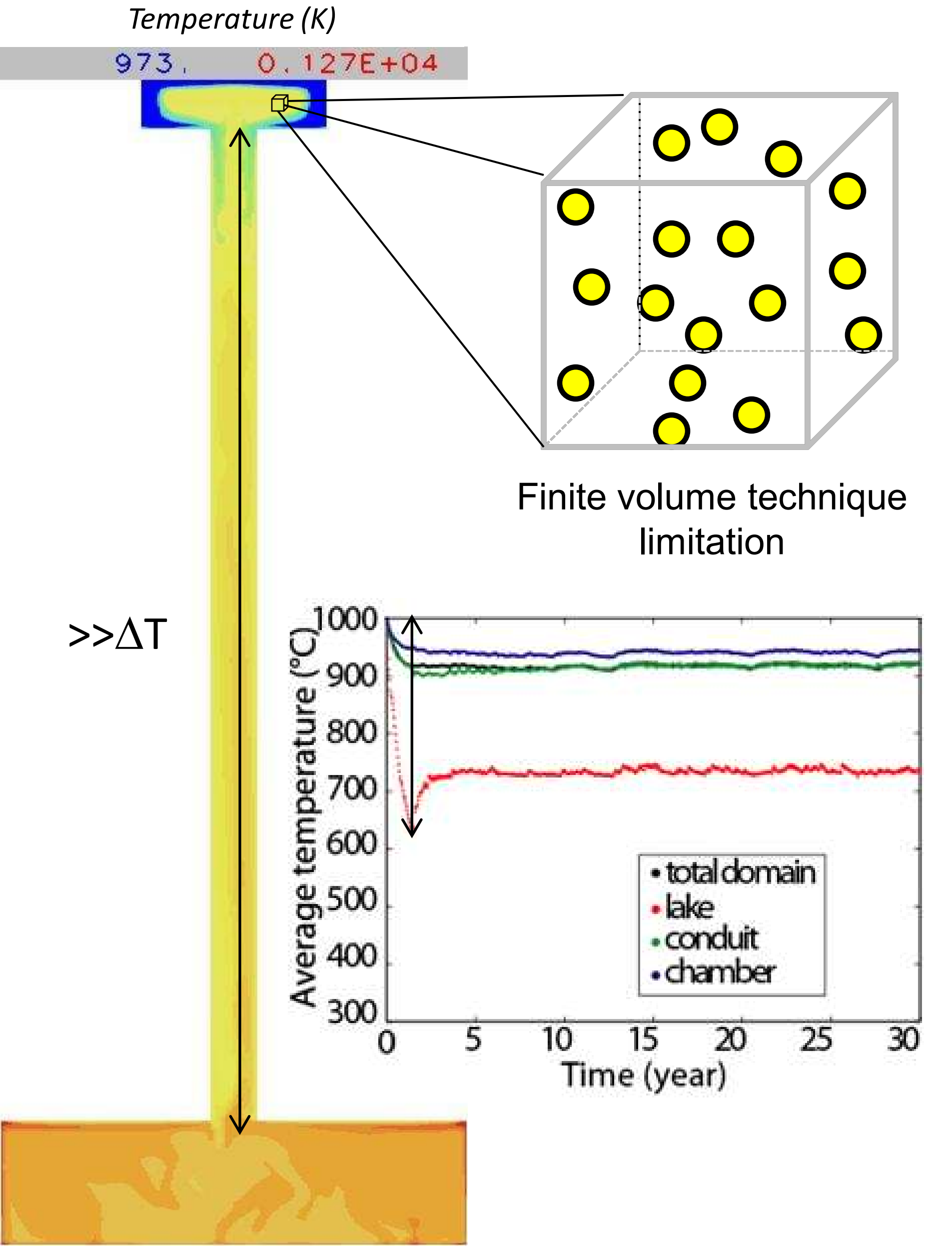


- Similar behaviour between bi-phase and mixture (central instability, average temperature).
- A 10-m conduit diameter allows keeping a sustained convection in the conduit and consequently higher temperatures in the lake.
- We can simulate the steady-state convection of the lake.

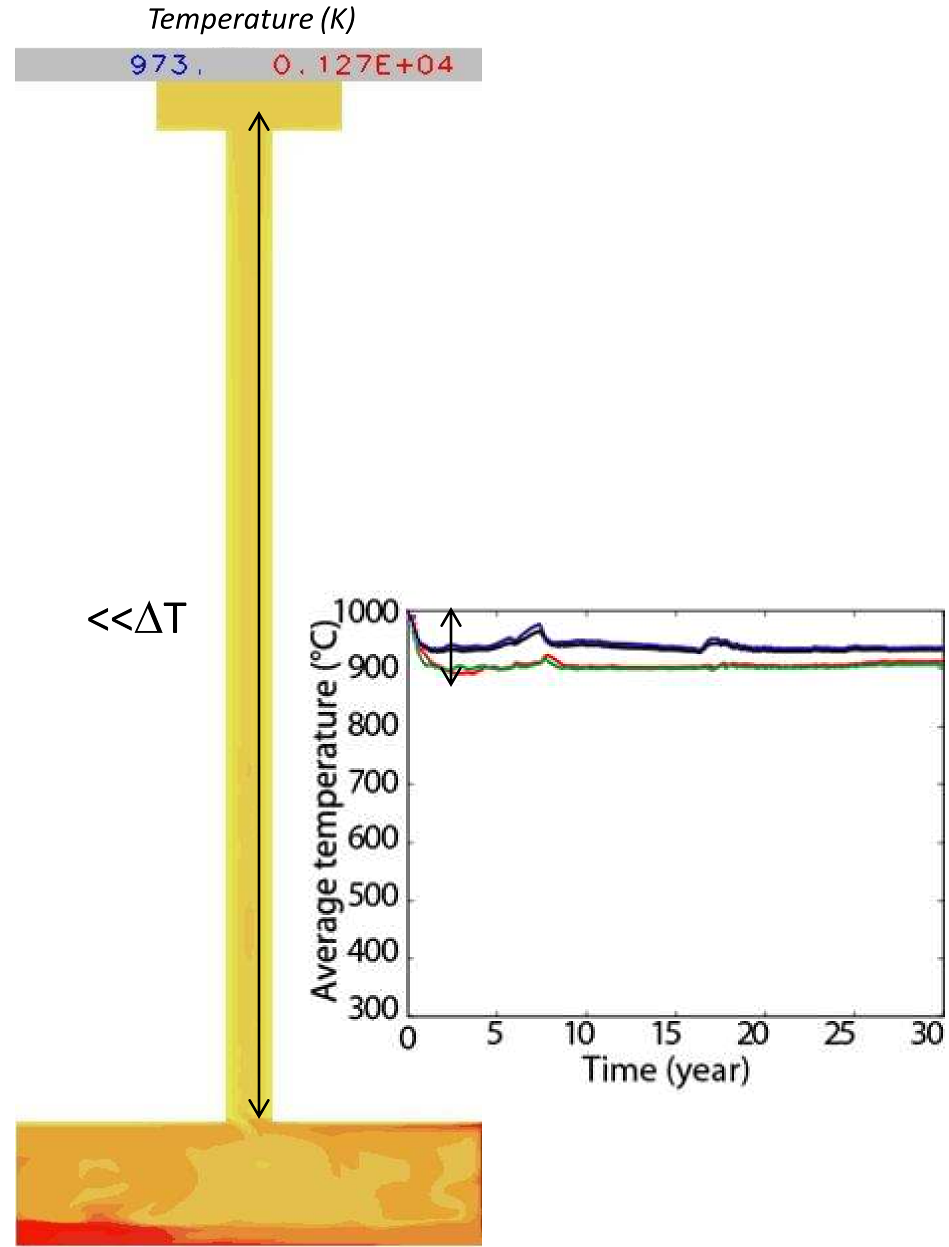
2. Numerical modeling: role of crystals

Bi-phase (closed vs. open system)

4 *Closed system (Year 30)*

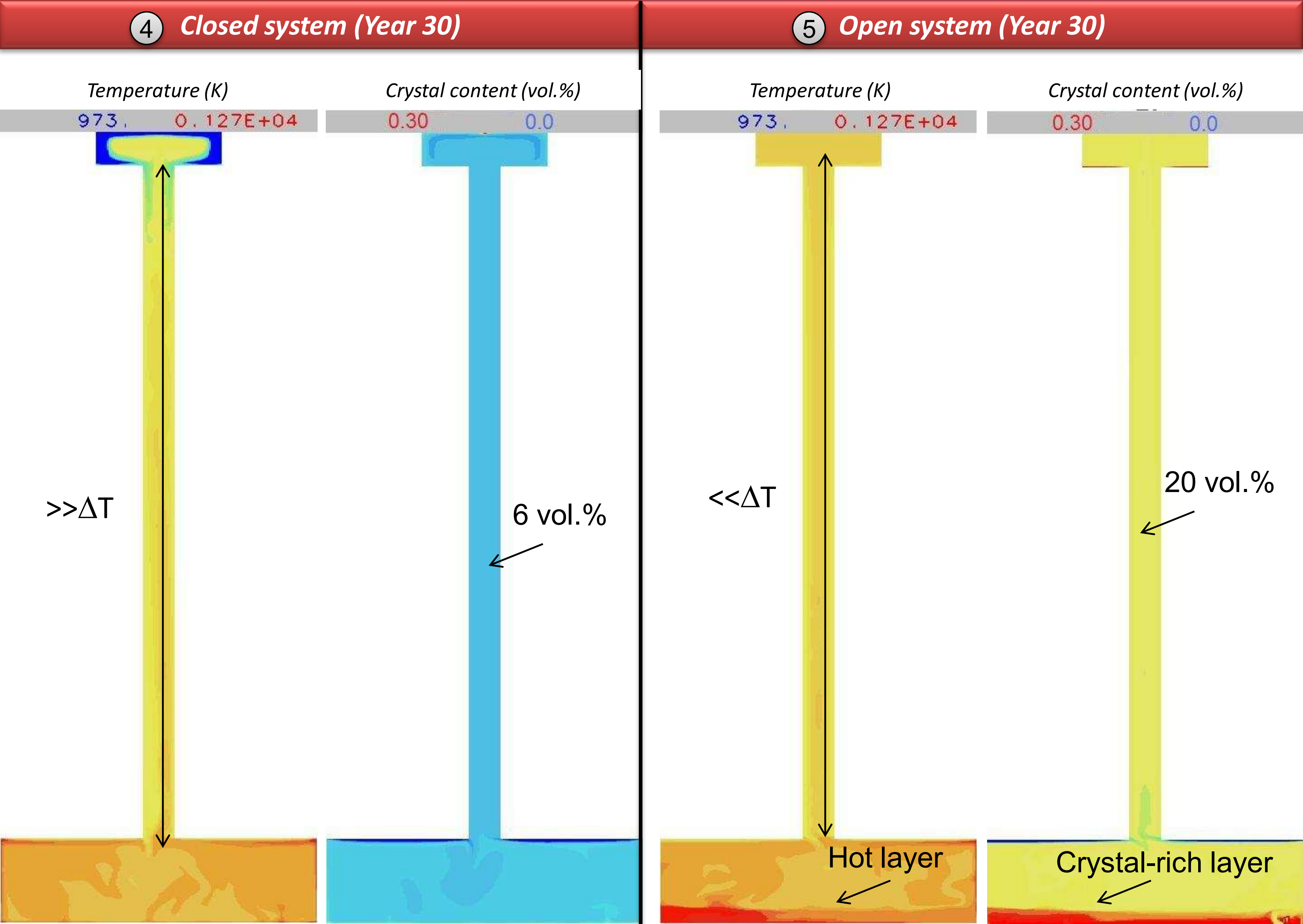


5 *Open system (Year 30)*



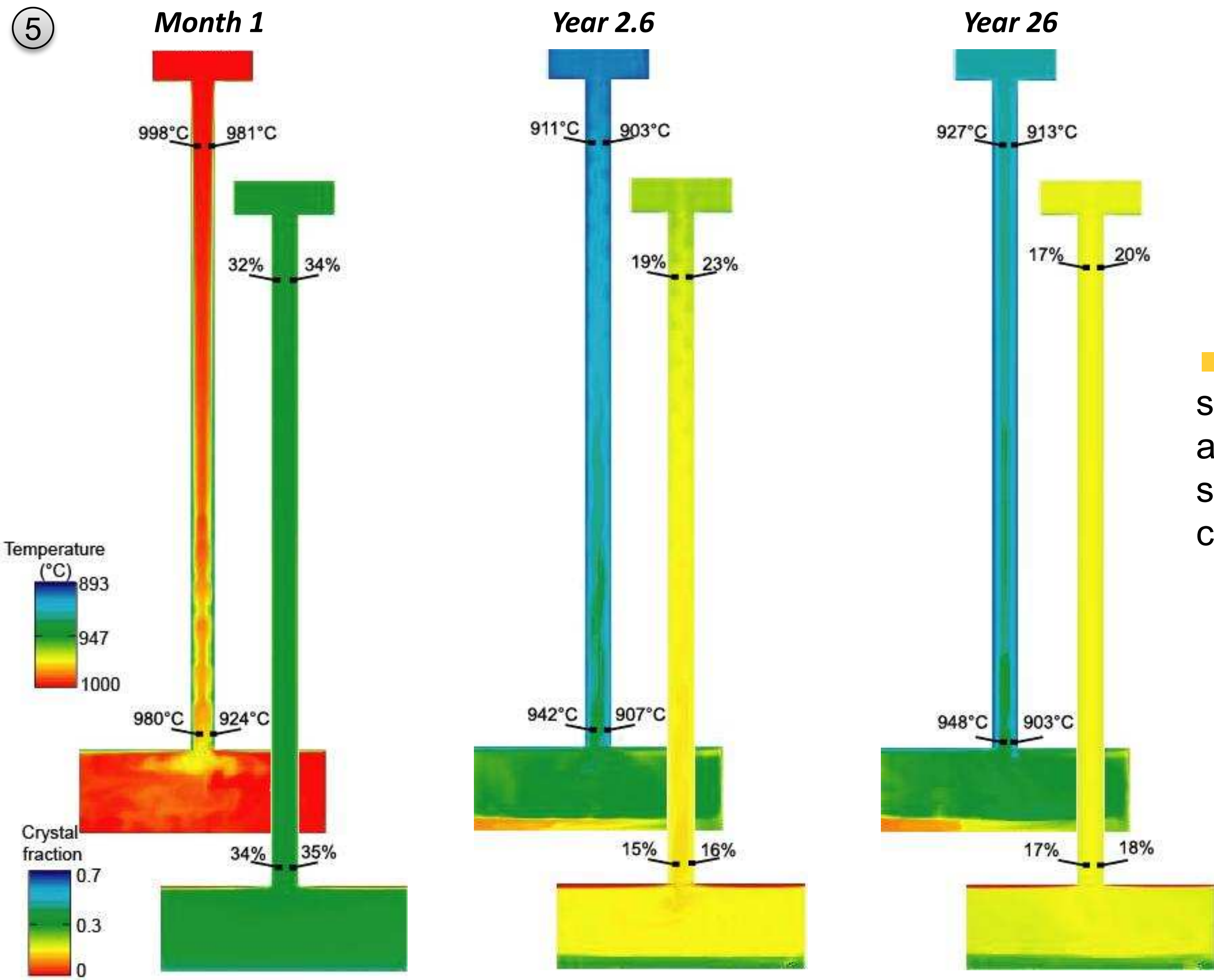
2. Numerical modeling: role of crystals

Bi-phase (closed vs. open system)



2. Numerical modeling: role of crystals

Bi-phase (open system)



ΔT: 8-17°C

Cristallinity: 17-34 %

- 20% of crystals in suspension represents a balance between sedimentation and convection.

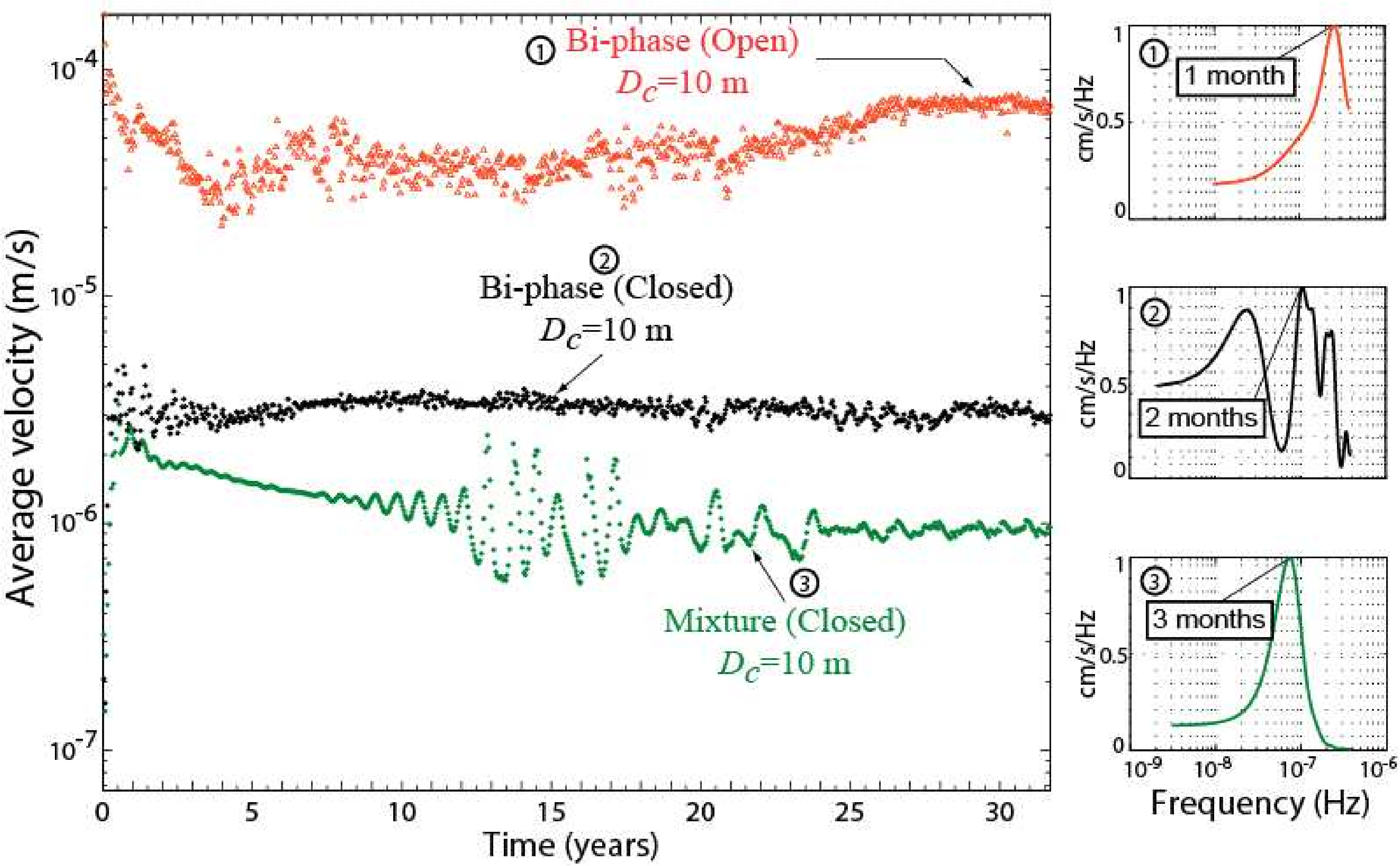
ΔT: 35-56°C

Cristallinity:15-35 %

ΔT=65°C (*Sweeney et al, 2008*)
ΔT<120°C (*Calkins et al, 2008*)
} Good agreement with our results

The smaller the crystals the more they remain in suspension

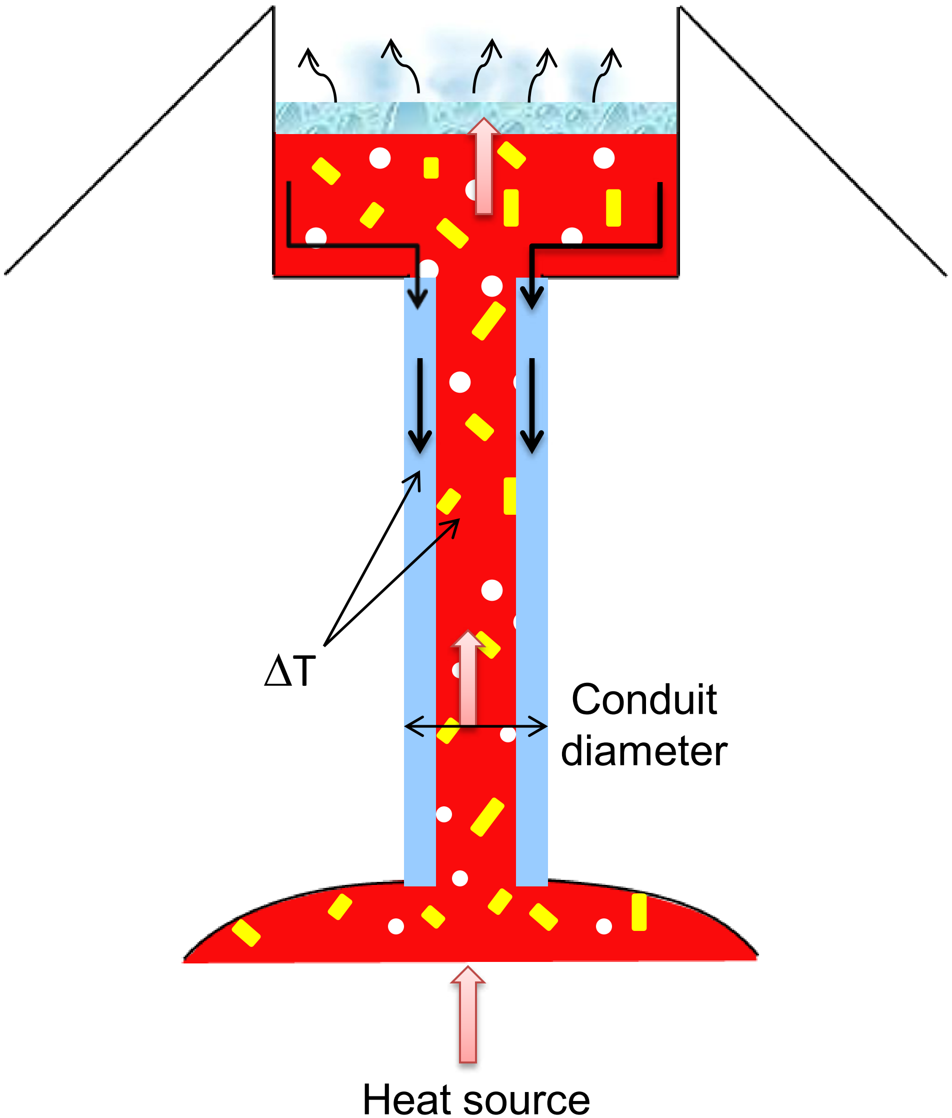
Surface velocities and frequency



- Velocities are 3 orders of magnitude lower than the observed values of 10^{-1} m/s
- Period is in the order of month far from the observed period of 10 min.

- Open system shows a higher convective rate

2. Numerical modeling: role of crystals



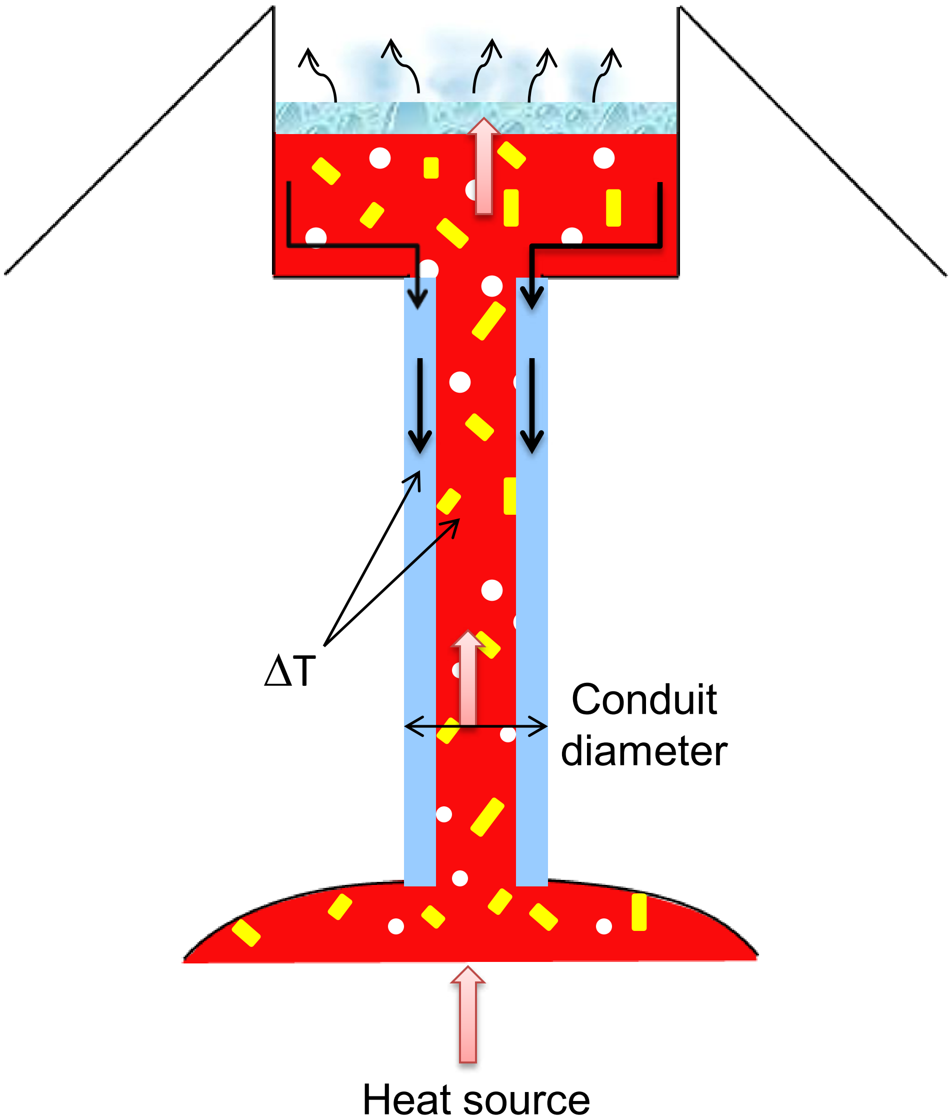
Conclusions

1. Can we simulate the convective regime with fluid dynamics?
   (1) Validation
   (2) Steady-state regime of Erebus.

2. Which conduit diameter is enough to sustain convection in Erebus?
   At least 10-m conduit diameter.

3. Can crystals be part of the melt?
   Bi-phase and mixture simulations show similar behaviour in terms of:
   (1) central instability behaviour
   (2) temperature evolution
   (3) surface velocities.

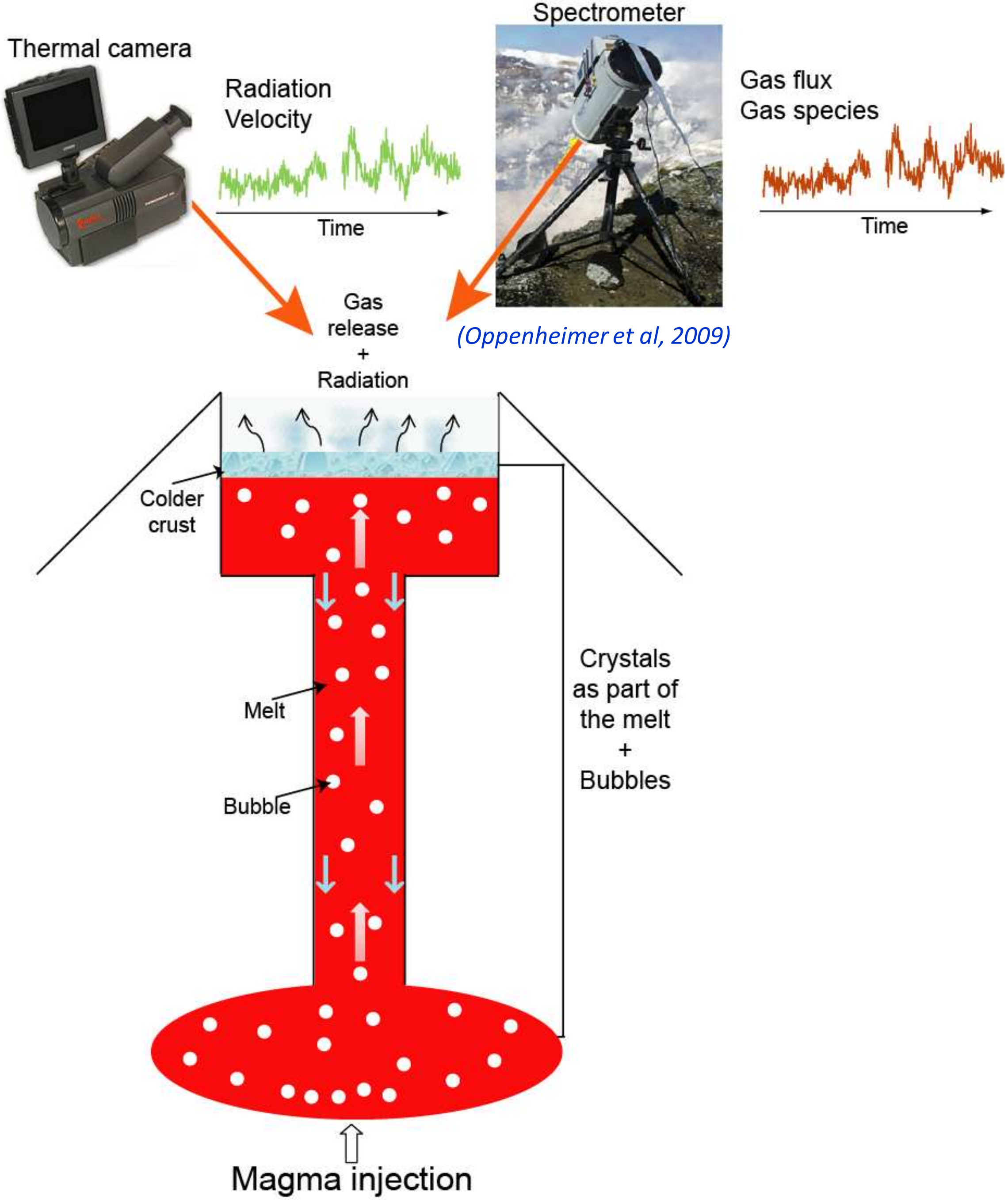
4. Which ΔT and crystal content characterize convective currents?
   (1) A maximum ΔT=56°C
   (2) A maximum Δ[xtal]=4 vol.%



5 Role of crystals

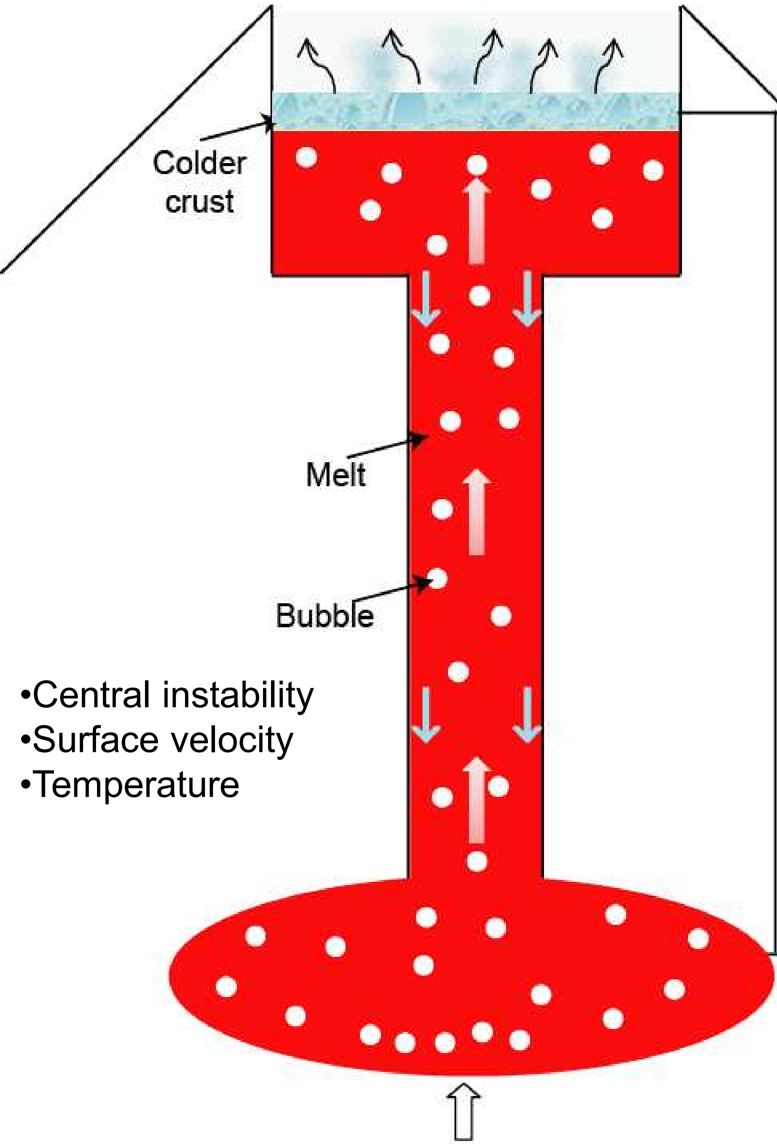
(1) 20 vol.% of crystal remain in suspension.
(2) They enhance the convective rate (amplitude and frequency); however those are still lower than observed values.

2. Numerical modeling: role of bubbles



Hypothesis

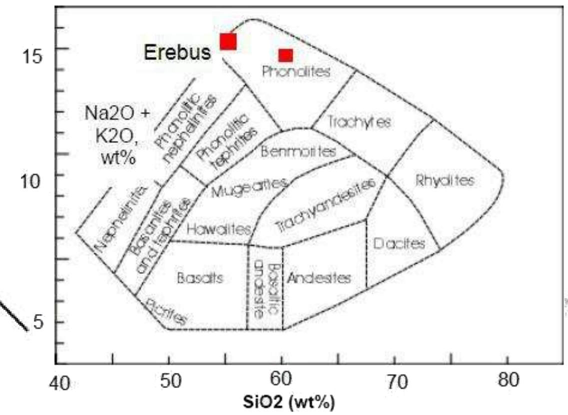
Open system



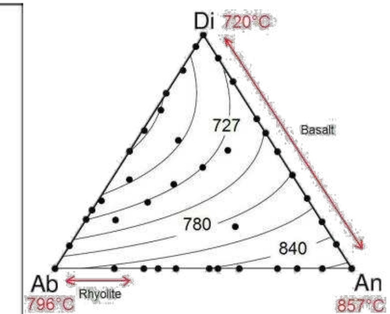
Magma injection

Crystals
as part of
the melt
+
Bubbles

•Central instability
•Surface velocity
•Temperature



•Variable bulk composition (as function of H_2O) (*Kelly et al, 2008*).
•Constant thermodynamical properties (e.g., heat capacity, conductivity, initial density) (*Mastin and Ghiorso, 2000*)
•Bulk viscosity as function of T (*Giordano et al, 2008*) ->30% crystals (*Krieger and Dougherty, 1959*) and H_2O.

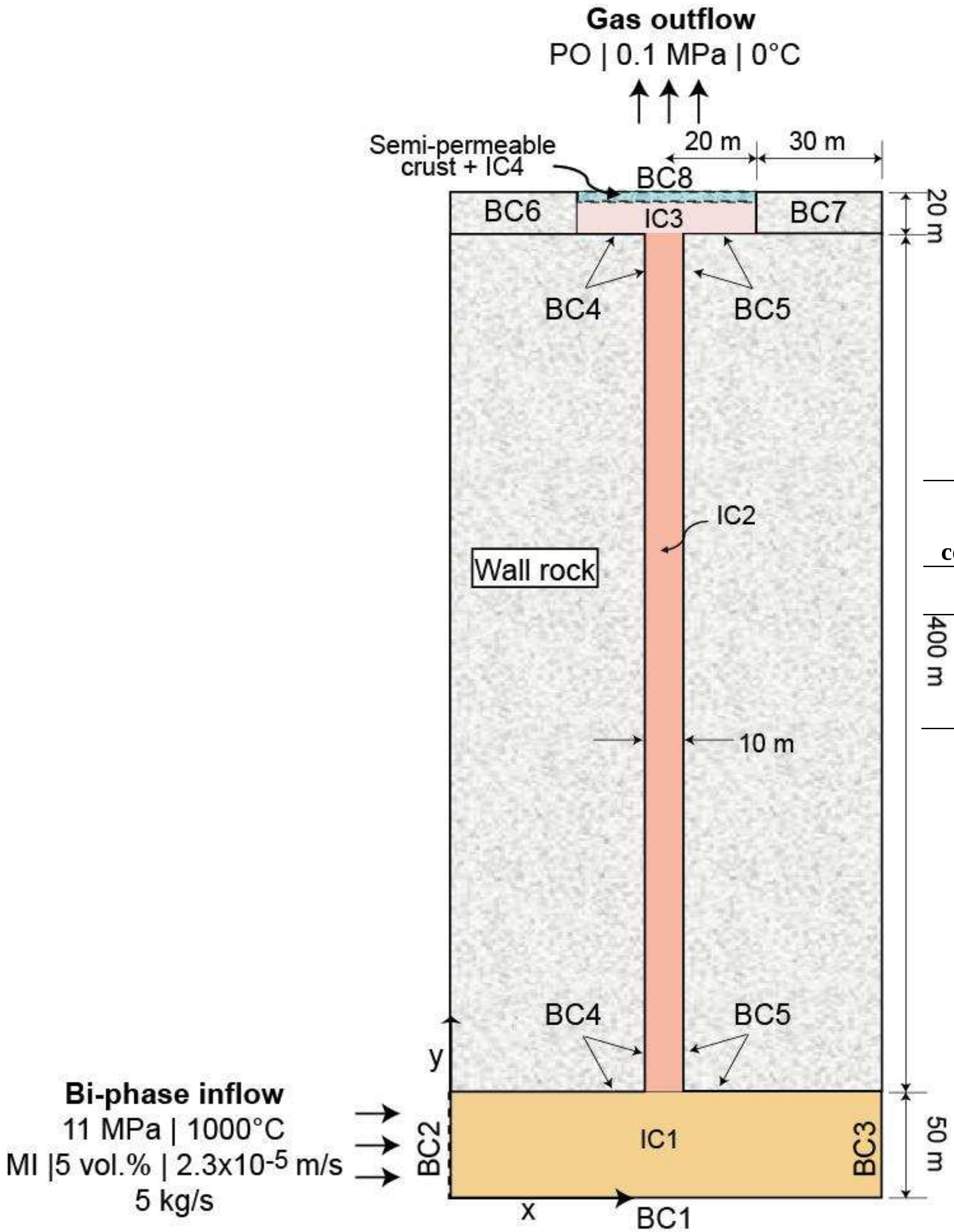


Glass transition
of 700°C
(*Russel and Giordano, 2005; Molina et al, 2012*)

Bubbles of H_2O
with thermodynamical
properties *(Ochkov, 2009)*

•Only H_2O
•Sphere
•Rigid
•Variable diameter
•Grow by expansion and diffusivity
•Non coallescence
•Non deviatoric stress
•Fixed BND 10^{11}
(Pers. comm. Schipper)

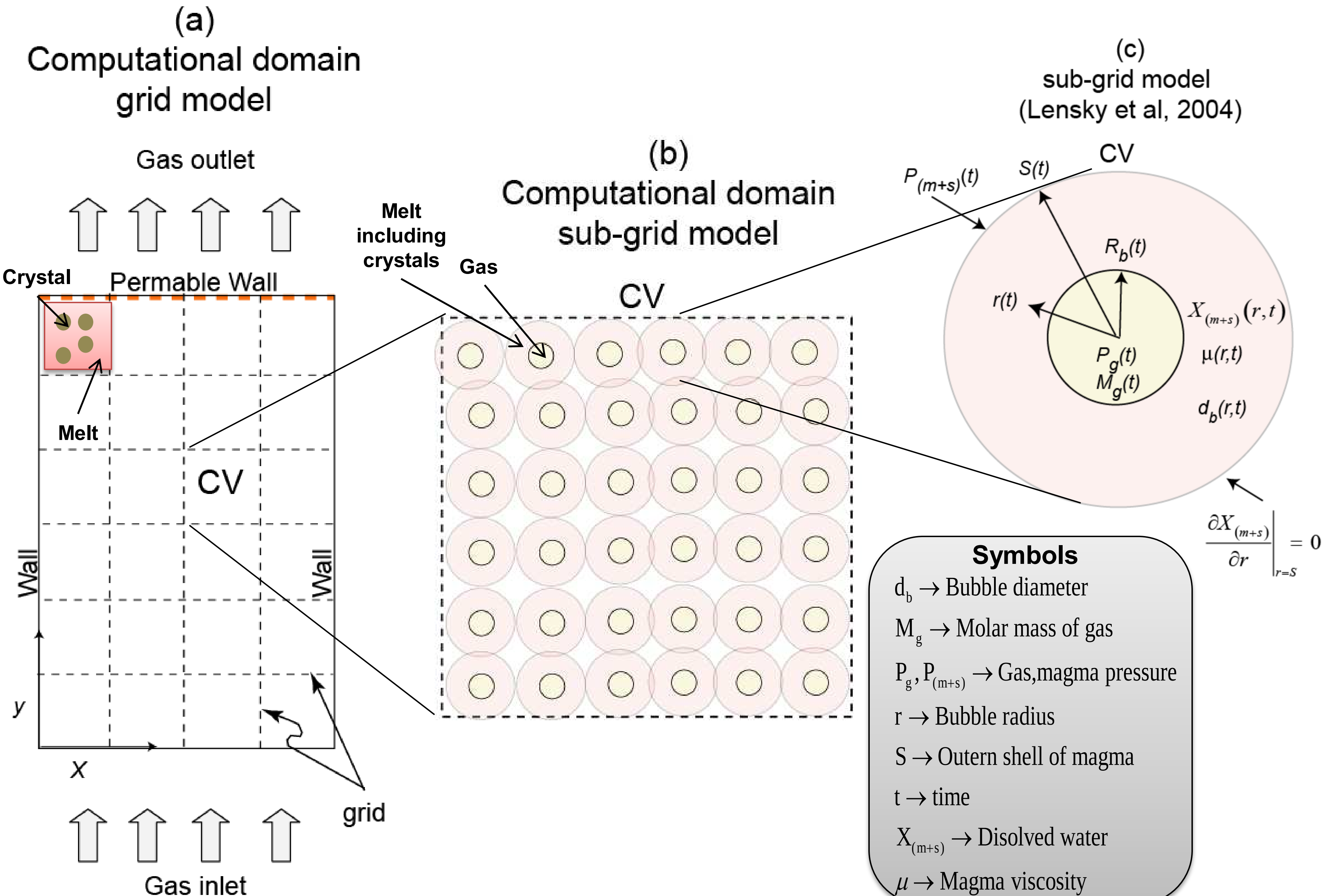
Hypotheses



Initial condition	Region of magma system	Mixture fraction	Bubble fraction	Temperature (°C)
		$\varepsilon_{(m+s)}$	ε_g	T_m
IC1	Chamber	0.999	0.001	1000
IC2	Conduit	0.950	0.050	1000
IC3	Lake	0.900	0.100	1000
IC4	Top of the lake	0.999	0.001	700

Crust:
Permeability 10^{-13} m^2
Gas velocity 10^{-6} m/s

Sub-grid model and expansion of bubbles



Symbols

- $d_b \rightarrow$ Bubble diameter
- $M_g \rightarrow$ Molar mass of gas
- $P_g, P_{(m+s)} \rightarrow$ Gas,magma pressure
- $r \rightarrow$ Bubble radius
- $S \rightarrow$ Outern shell of magma
- $t \rightarrow$ time
- $X_{(m+s)} \rightarrow$ Disolved water
- $\mu \rightarrow$ Magma viscosity

Continuity (fluid+gas)

Density

$$\rho \begin{cases} \rho_{(m+s)} = \varepsilon_s \rho_s + \varepsilon_m \left\{ \rho_o \left[1 - \alpha \left(T_{(m+s)} - T_o \right) \right] \right\} \\ \rho_g = \dfrac{P_g M_g}{G T_g} \end{cases}$$

Reaction rate

$$\frac{\partial X_{(m+s)}}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D_{H_2O} \frac{\partial X_{(m+s)}}{\partial r} \right)$$

Water diffusivity

$$D_{H_2O} = \exp\left[-11.924 - 1.003 \ln\left(X_{(m+s)} \right) \right] \exp\left\{ \frac{-\exp\left[11.836 - 0.139 \ln\left(X_{(m+s)} \right) \right]}{G T_{(m+s)}} \right\}$$

Freda et al, 2003

Momentum (fluid+gas)

Bulk viscosity

$$\mu \begin{cases} \mu_g = 0 \\ \mu_{(m+s)} = \mu_m \left(1 - \dfrac{\varepsilon_s}{1 - \varepsilon_m^*} \right)^{-[\eta](1-\varepsilon_m^*)} \end{cases}$$

$f(H_2O)$

Giordano et al, 2008
Krieger and Dougherty, 1959

Stress tensor

$$\overline{\overline{S}} \begin{cases} \overline{\overline{S}}_{(m+s)} = P_{(m+s)} \overline{\overline{I}} + \overline{\overline{\tau}}_{(m+s)} \\ \overline{\overline{S}}_g = P_g \overline{\overline{I}} \end{cases}$$

By simplifying Lensky et al, 2004

$$P_g = P_{(m+s)} + \frac{4\sigma}{d_b} + \frac{4\dot{d}_b}{d_b} \mu_{(m+s)} \left(1 - \varepsilon_g \right)$$

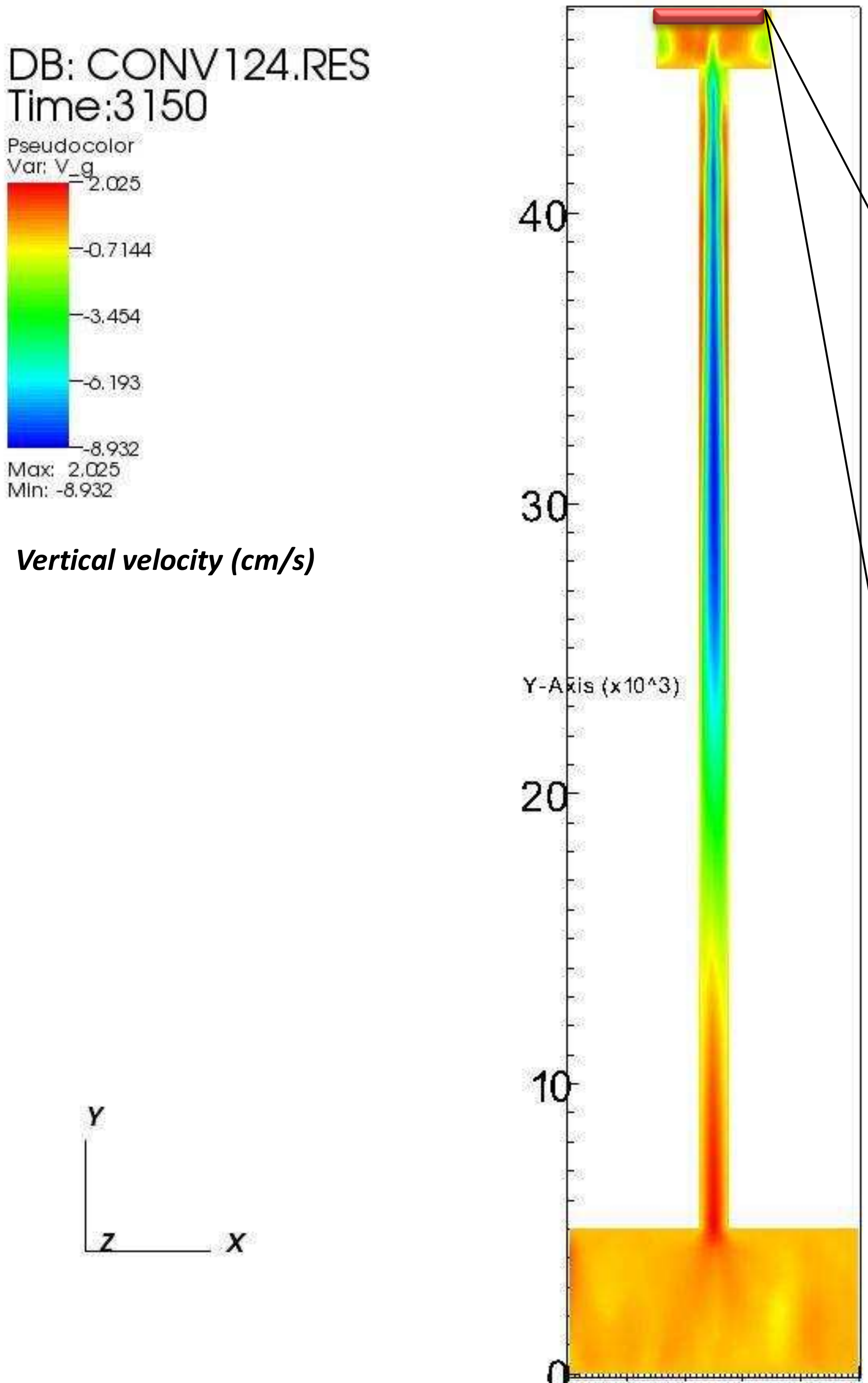
Drag

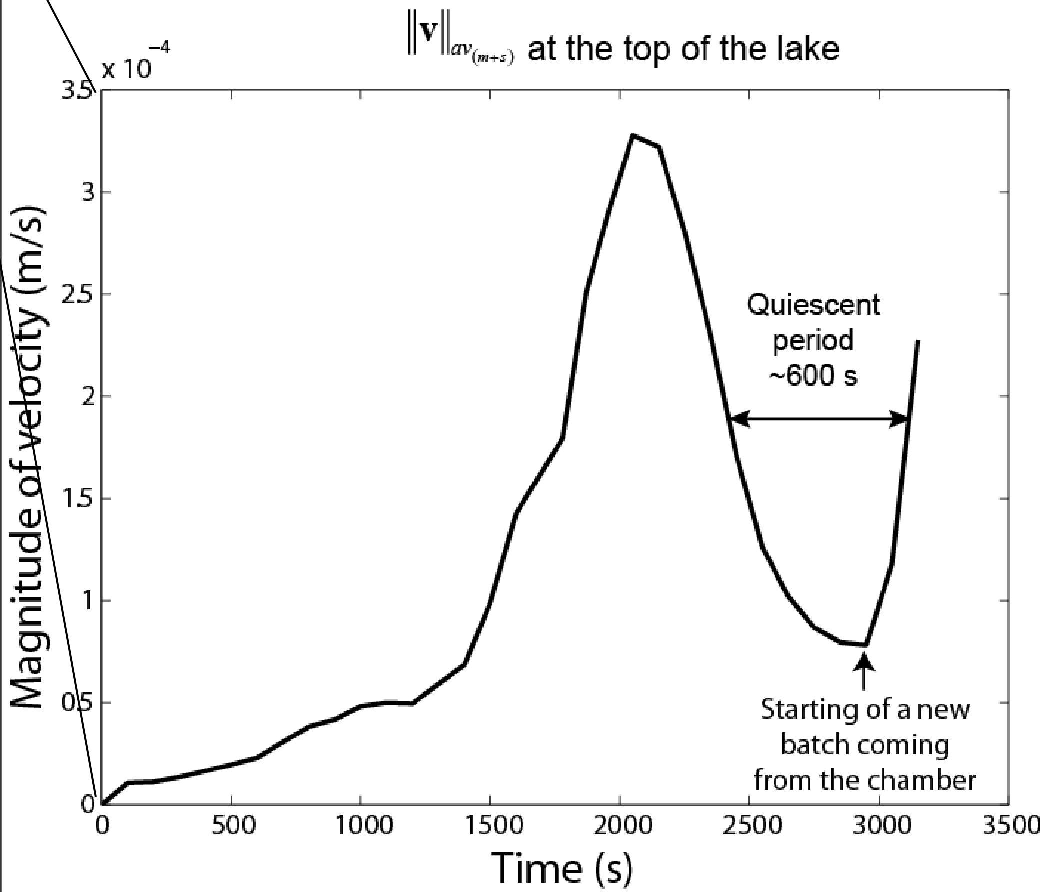
Joshi et al, 2001

Symbols

- $d_b \rightarrow$ Bubble diameter
- $D_{H_2O} \rightarrow$ Water diffusivity
- $G \rightarrow$ Universal gas constant
- $M_g \rightarrow$ Molar mass of gas
- $P_g, P_{(m+s)} \rightarrow$ Gas,magma pressure
- $P \rightarrow$ pressure depth
- $r \rightarrow$ Bubble radius
- $T_{(m+s)}, T_o \rightarrow$ Mixture temperature
- $X_{(m+s)} \rightarrow$ Disolved water
- $\sigma \rightarrow$ Surface tension
- $\varepsilon_s, \varepsilon_s \rightarrow$ Crystal/melt fraction
- $\rho_{(m+s)}, \rho_g \rightarrow$ Mixture/gas density
- $\mu_g, \mu_{(m+s)} \rightarrow$ Gas, mixture viscosit
- $\overline{\overline{I}}, \overline{\overline{\tau}}_{(m+s)} \rightarrow$ Identity, viscous tensor

General results

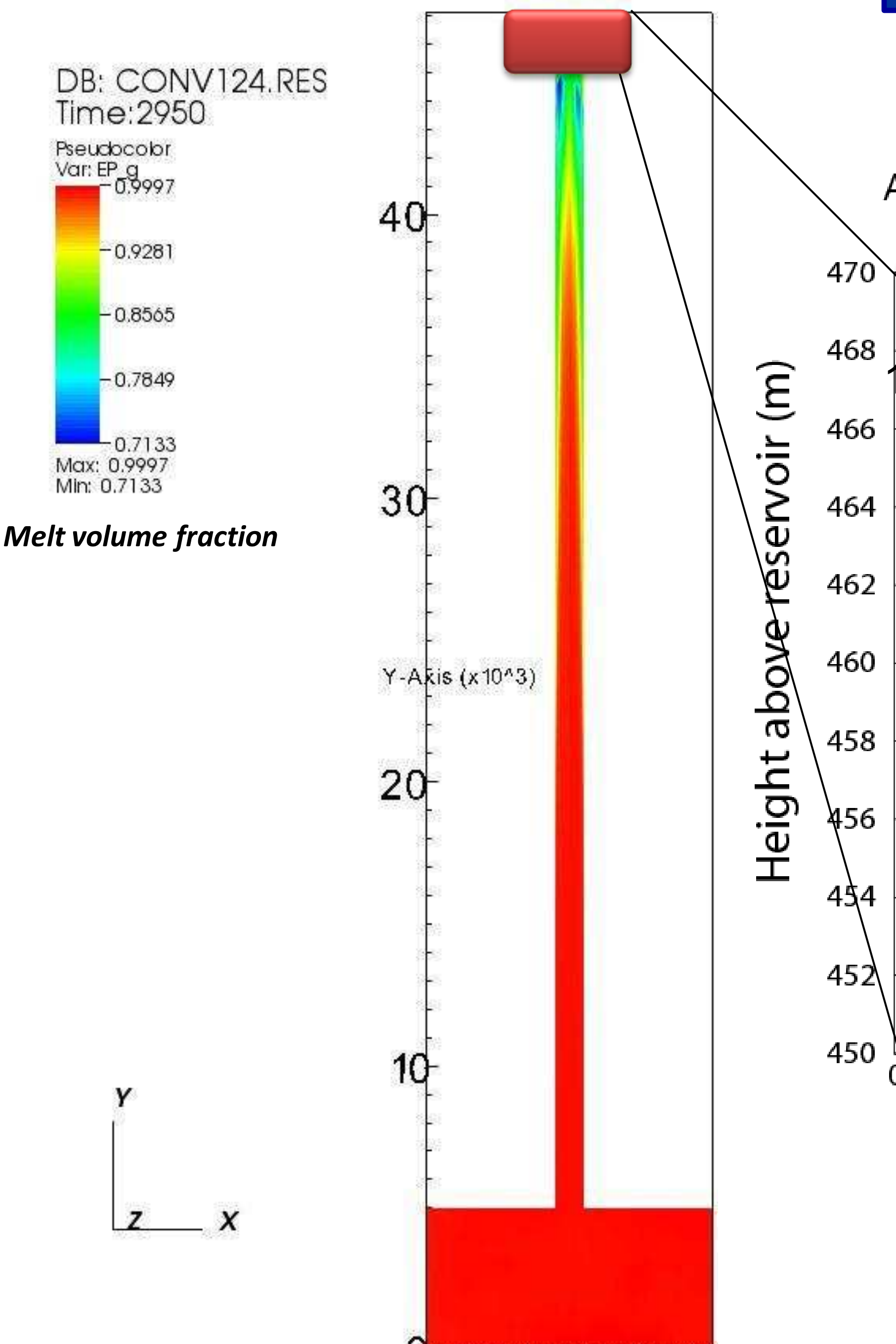


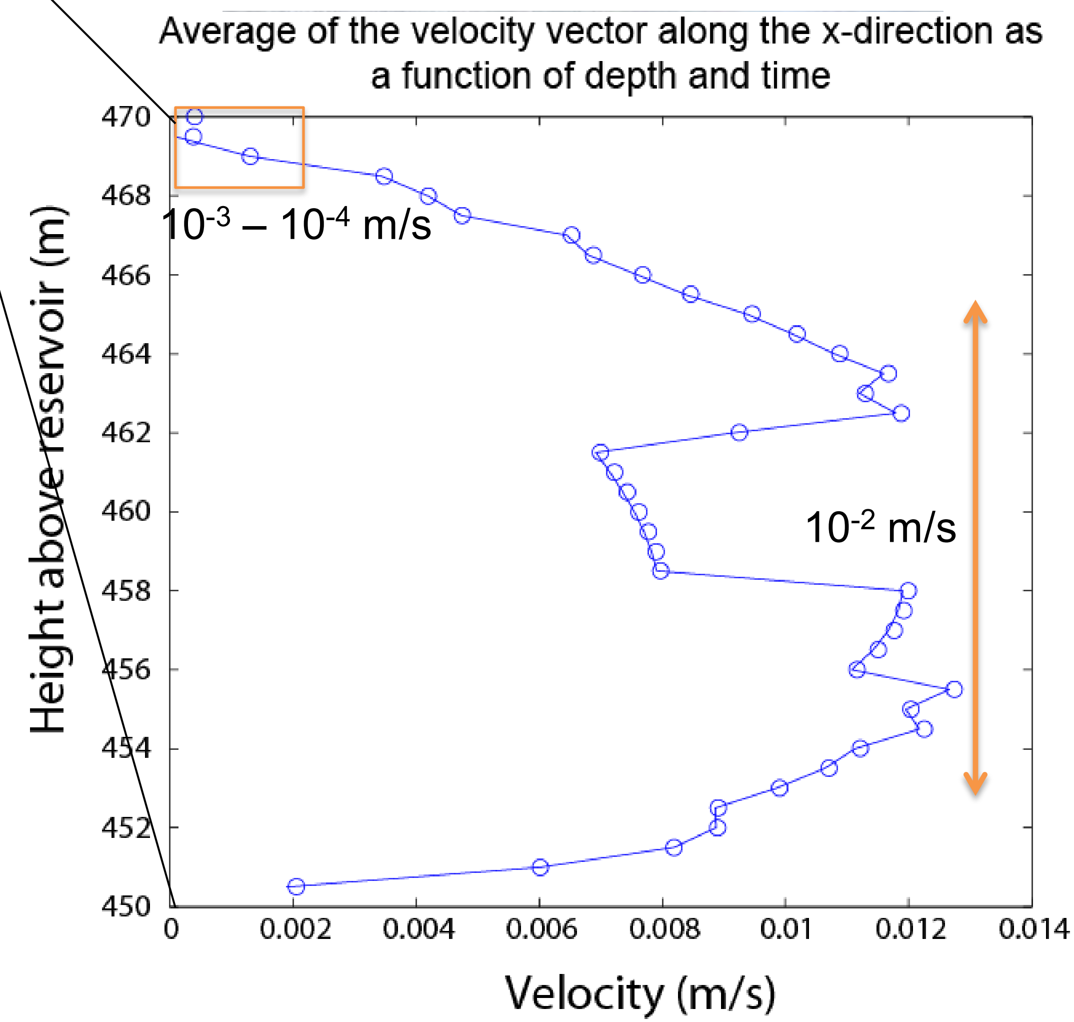


Observed cycle of 600 s (10 min)

(Oppenheimer et al, 2009)

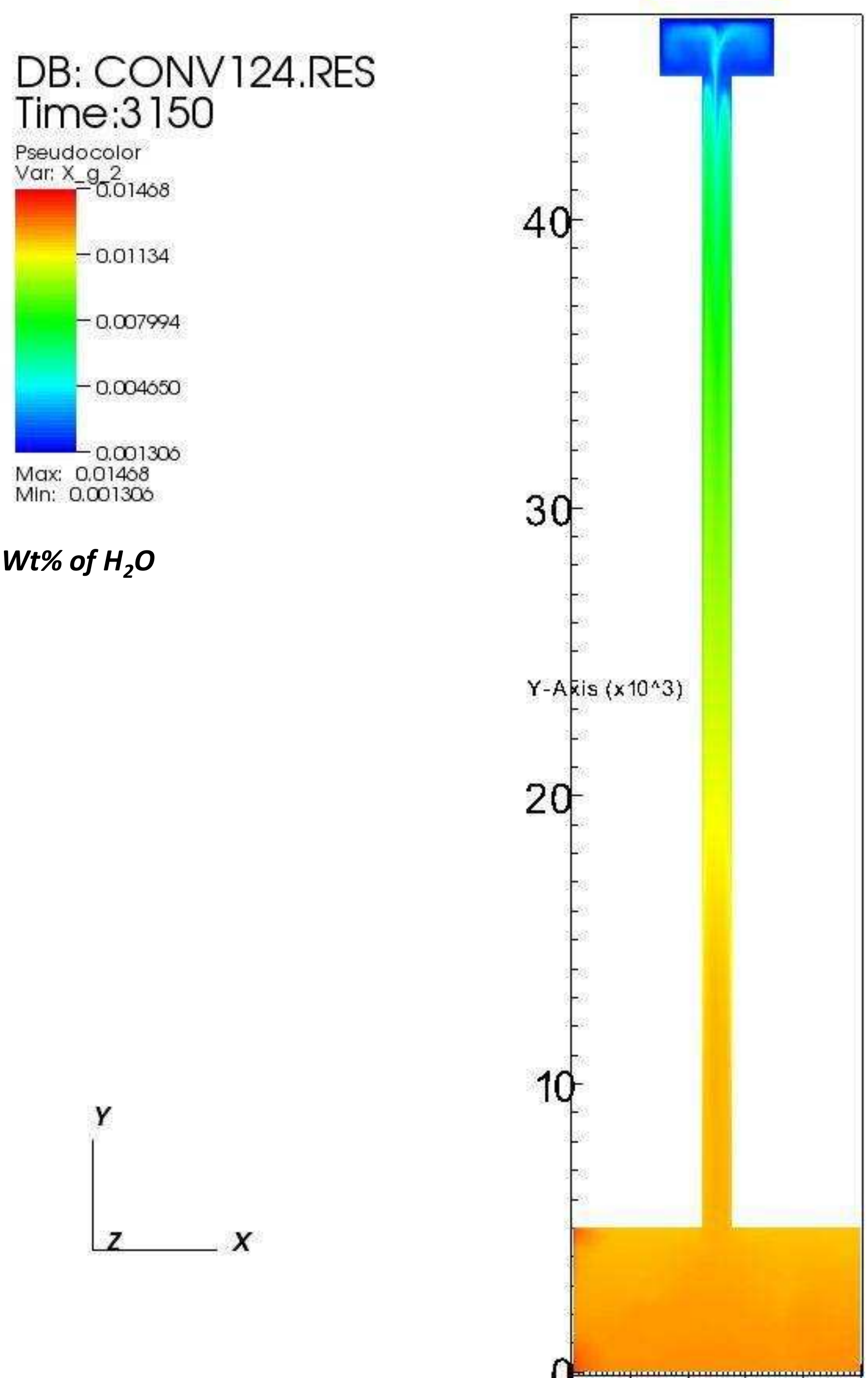
General results



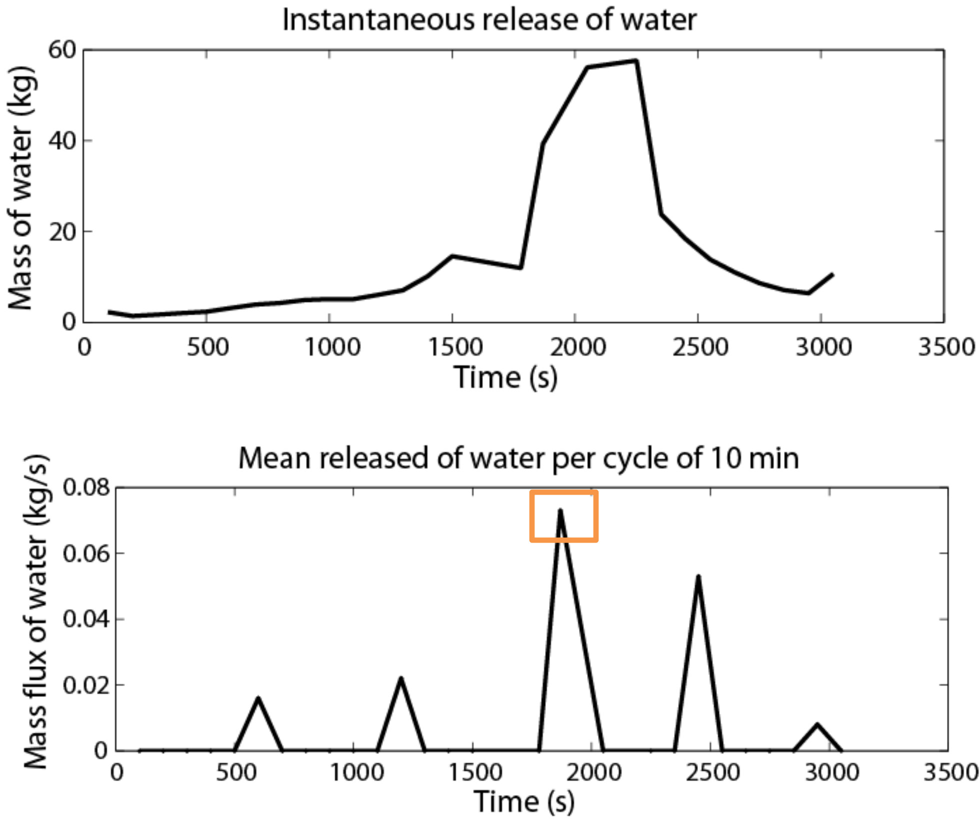


Observed velocity
10^{-1} m/s *(Oppenheimer et al, 2009)*

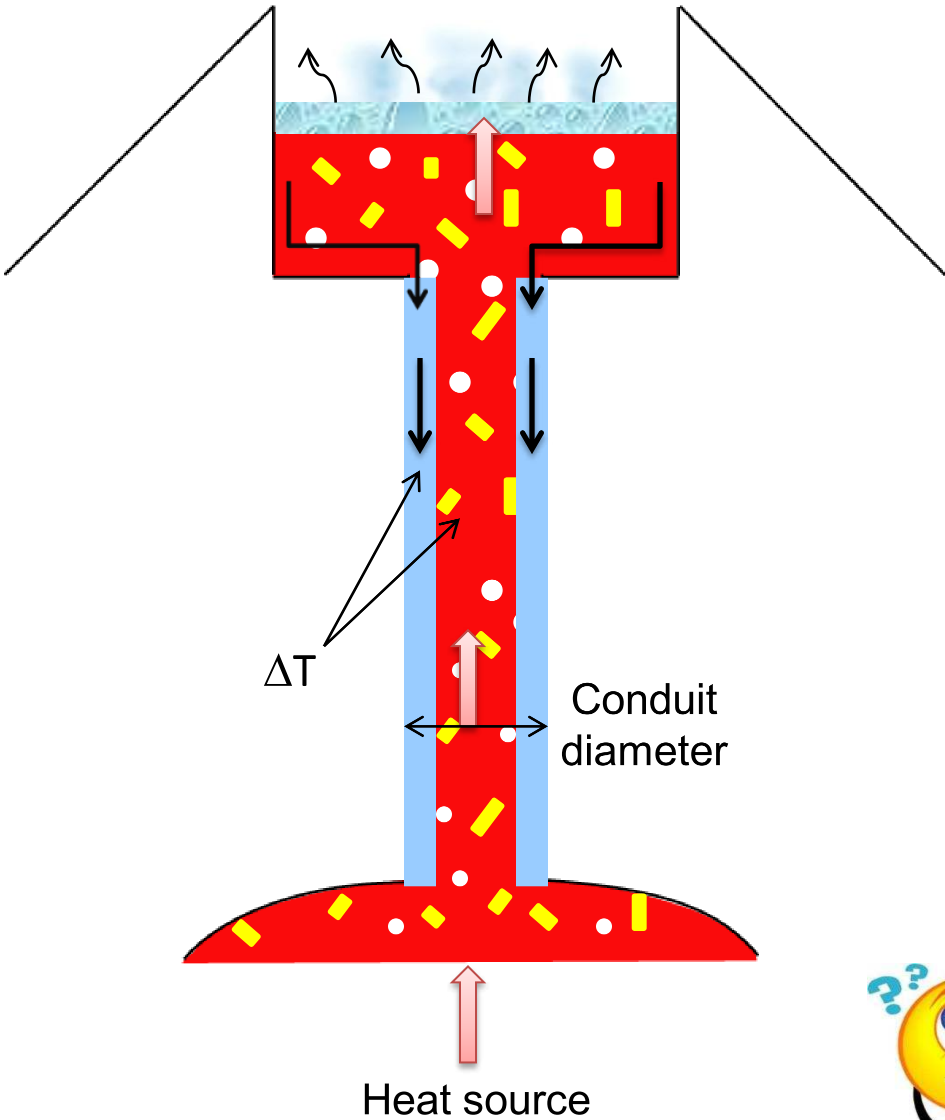
Results



Wt% of H_2O



Observed released water of 7 kg/s
(Oppenheimer et al, 2009)



6 Explain the role of bubbles.

Our simulation suggest that magma can rise by intermittent batches of magma.

<u>Bubbles increase velocity rate</u> in one order of magnitude but this rate remains two orders of magnitude lower than observed one.

The gas water flux output is two orders of magnitude lower compared to observations.

In order to approach to reality we need to work on the following parameters:

Change the permeability of crust.
Change the rate of recharge.

Numerical

- This is the first study aiming at understanding the convection of the whole magma system with the MFIX model.
- We can simulate the permanent convection in Erebus provided the conduit be large enough.
- Crystal can be considered as part of the melt, however their presence (and more so the presence of bubbles) enhances the modeled velocities at the surface of the lava lake. Those velocities are still 2 orders of magnitude lower than the observed ones.
- Simulations including bubbles can reproduce 10 min period, but we need to reach more time of simulation.

A direction for future work: include the coalescence process.

Remerciements

Alain Burgisser (ISTO)

Clive Oppenheimer (Cambridge University)
Geneviève Brandeis (IPGP)
Timothy Druitt (Université de Clermont Ferrand)
Marielle Collombet (Université de Savoie)
Jean-Louis Bourdier (Université d'Orléans)
Hiroyuki Kumagai (NIED)

Tous mes collègues de l'ISTO (Manu L-T, Juan A, Bruno S, Marina A, Caroline M, Iada G, Michel P, Gabriel C, Fabrice G, Anita C, Michael L, Ida, Remi C), et ceux qu'on passé par ici (Caroline B-de-M, Simona M, Ian S, Jonhatan C, Wimm D, Antonio C, Claudia A, Yves, Gaelle)

Mes amis de partout (Hélène H, Lina O, Leire D-C, Nani R, Juan A, Aurelien C, Christophe T, Christelle Z, Gerardo S, Nico, de Amérique du Sud)

Ma famille (Mama, Don F, Marie-Louise, Claire, hermanitos, Nathalie, PJ, JY, Ben, Eli, Diego)