Multi-Label Classification using a Map Between Sparse Representations: Application to Retinal Blood Vessel Segmentation

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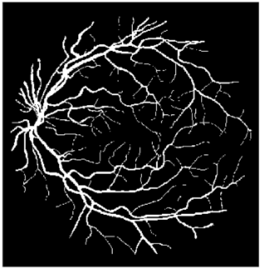
Retinal Blood Vessel Segmentation (RBVS)

Fundus image



Segmentation of blood vessels

Why?

- Eye vascularization is an important risk biomarker in a large number of diseases, *e.g.* diabetic retinopathy, age-related macular degeneration and glaucoma.
- The eye shares neural and vascular similarities with the brain. Eye vascularization offers a direct window to cerebral pathology.

Learning RBVS

Learning to segment

Assume that we have a training set $L = \{(I_i, S_i)\}_{1 \leq i \leq n}$, where

- $I_i \in \mathbb{R}^N$ is a (vectorized) fundus image
- $S_i \in \{0,1\}^N$ is an expert segmentation of I_i.

Learning RBVS means using L to construct a mapping $\mathcal{S} : \mathbb{R}^N \to \{0,1\}^N$ that allows to infer the segmentation of new, unseen fundus images.

Training set $\xrightarrow{\text{Learn}}$ $\underbrace{\mathcal{S} : \mathbb{R}^N \to \{0,1\}^N}_{\text{Segmentation function}}$

Learning by Feature Extraction and Empirical Risk Minimization

Patches

- We work at the level of patches, that is small $\sqrt{m} \times \sqrt{m}$ image subsets.
- The training set consists of two matrices:
  - $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_n] \in \mathbb{R}^{m \times n}$, whose columns are (pre-processed) image patches
  - $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_n] \in \{0, 1\}^{m \times n}$, whose columns are the corresponding expert segmentations.

Feature Extraction

- Instead of working with raw patches, we try to find good *features* that will ease segmentation.
- Feature extraction is given by a function (usually nonlinear) $F : \mathbb{R}^m \to \mathbb{R}^p$.
- The features of a test patch will serve either to infer the label of its central pixel, or the labels of all its pixels.

ERM

1. Choose a class $\mathcal{H}$ of plausible segmentation functions.
2. Choose an $S \in \mathcal{H}$ that "works well" on the training set $(\mathbf{X}, \mathbf{Y})$, that is, that minimizes an empirical risk.

This talk: feature extraction by sparse representation in a redundant learned dictionary

Sparse Dictionary Learning

Principle

Learn a redundant dictionary $(\mathbf{d}_i)_{1\geq i\geq p}$, $p > m$, such that each training patch $\mathbf{x}_j \in \mathbb{R}^m$ can be represented *sparsely* represented as a linear combination of the $\mathbf{d}_i'$s.

A Combinatorial Formulation

$$\mathbf{D}^*, \mathbf{A}^* \leftarrow \min_{\mathbf{D}\in\mathbb{R}^{m\times p}, \mathbf{A}\in\mathbb{R}^{p\times n}} \Big[\mathcal{R}(\mathbf{X}, \mathbf{D}, \mathbf{A}) = \lambda \sum_{i=1}^{n} \|\mathbf{x}_i - \mathbf{D}\mathbf{a}_i\|_2^2 + \|\mathbf{a}_i\|_0\Big],$$

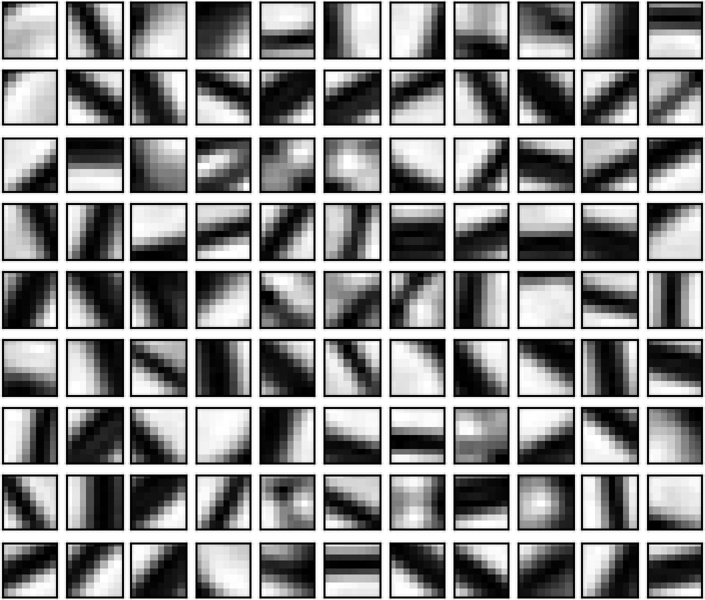
where $\|\mathbf{x}\|_0$ is the number of non-zero entries of $\mathbf{x}$.

A (partially) Convex Formulation

$$\mathbf{D}^*, \mathbf{A}^* \leftarrow \min_{\mathbf{D}\in\mathbb{R}^{m\times p}, \mathbf{A}\in\mathbb{R}^{p\times n}} \Big[\mathcal{R}(\mathbf{X}, \mathbf{D}, \mathbf{A}) = \lambda \sum_{i=1}^{n} \|\mathbf{x}_i - \mathbf{D}\mathbf{a}_i\|_2^2 + \|\mathbf{a}_i\|_1\Big]. \quad (1)$$

In both versions, the columns of $\mathbf{D}$ should be constrained to have unit norm.

An Example of Learned Dictionary



RBVS by Sparse Coding then Classifier (SCTC)

Training

1. Using Eq. (1), learn a redundant dictionary $\mathbf{D}$ that sparsely encodes each training patch $\mathbf{x}_i$.
2. Train a classifier $\mathcal{C}$ (*e.g.* random forest) on the sparse codes (the $\mathbf{a}_i'$s) of each $\mathbf{x}_i$ in order to infer the label of the central pixel.

Testing

1. Extract all patches of the test image.
2. Represent each test patch $\mathbf{x}$ as sparse linear combination of the columns of $\mathbf{D}$ by solving

$$\mathbf{a}^* \leftarrow \min_{\mathbf{a} \in \mathbb{R}^p} \lambda \|\mathbf{x} - \mathbf{D}\mathbf{a}\|_2^2 + \|\mathbf{a}\|_1. \tag{2}$$

3. Compute $\mathcal{C}(\mathbf{a})$ to obtain the label of the central pixel of $\mathbf{x}$.

Both the feature extraction and the classifier are learned. But feature extraction has been learned in an unsupervised way.

RBVS by Joint Dictionary and Classifier Learning (JDCL)

Learning

- Jointly learn
  - a dictionary **D** that sparsely encodes training patches
  - and a linear classifier **W** whose inputs are the sparse codes produced by **D**:

$$\mathbf{D}^*, \mathbf{A}^*, \mathbf{W}^* \leftarrow \min_{\mathbf{D},\mathbf{A},\mathbf{W}} \alpha \|\mathbf{L} - \mathbf{W}\mathbf{A}\|_F^2 + \beta \|\mathbf{W}\|_F^2 + \mathcal{R}(\mathbf{X}, \mathbf{D}, \mathbf{A}),$$

$\mathbf{L} = [\mathbf{l}_1, ..., \mathbf{l}_n] \in \mathbb{R}^{2\times n}$ is the label matrix where the vector $\mathbf{l}_i \in \mathbb{R}^2$ has binary entries indicating whether the center of the associated patch belongs to a vessel or not.

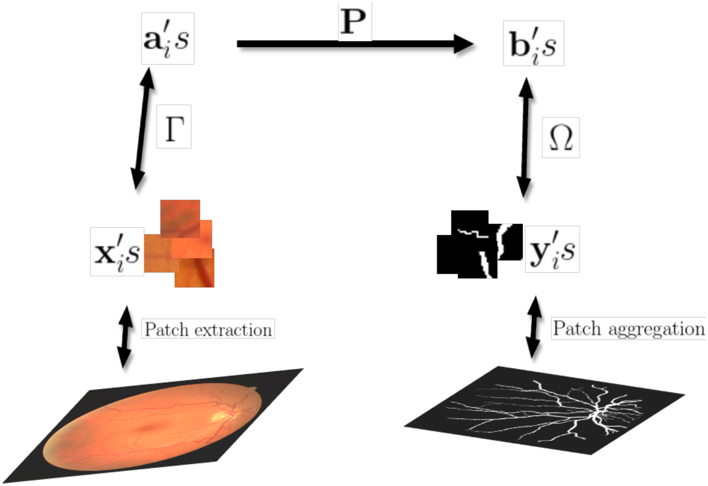
Testing

The label l of the central pixel of a query patch $\mathbf{x}$ is obtained using the following equation:

$$l = \underset{i\in\{1,2\}}{\text{argmax}} \langle \delta_i, \mathbf{W}^*\mathbf{a}^* \rangle,$$

where $\mathbf{W}^*$ is the previously learned classifier and $\mathbf{a}^*$ is the sparse code of $\mathbf{x}$ in $\mathbf{D}^*$.

Multi-Label Classification using a Map Between Sparse Representations



The Learning Stage

- Learning a label dictionary $\mathbf{\Omega}^*$:

$$\mathbf{\Omega}^*, \mathbf{B}^* \leftarrow \min_{\mathbf{\Omega} \in \mathcal{D}, \mathbf{B} \in \mathbb{R}^{p \times n}} \mathcal{R}(\mathbf{Y}, \mathbf{\Omega}, \mathbf{B}),$$

- Simultaneously learning a data dictionary $\mathbf{\Gamma}^*$ and a linear map $\mathbf{P}^*$:

$$\mathbf{\Gamma}^*, \mathbf{A}^*, \mathbf{P}^* \leftarrow \min_{\substack{\mathbf{\Gamma} \in \mathcal{D}, \\ \mathbf{A} \in \mathbb{R}^{p \times n}, \mathbf{P} \in \mathbb{R}^{p \times p}}} \mathcal{R}(\mathbf{X}, \mathbf{\Gamma}, \mathbf{A}) + \alpha \mathcal{C}(\mathbf{B}^*, \mathbf{P}, \mathbf{A}), \tag{3}$$

  with

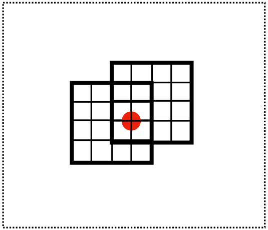
$$\mathcal{C}(\mathbf{B}^*, \mathbf{P}, \mathbf{A}) = \frac{1}{2} \|\mathbf{B}^* - \mathbf{P}\mathbf{A}\|_F^2 + \beta \|\mathbf{P}\|_F^2.$$

The Testing Stage

- Divide the input image into *overlapping* patches
- For each input patch $\mathbf{x} \in \mathbb{R}^m$, compute its segmentation map $\mathbf{s} \in \{0,1\}^m$ as follows

$$\mathbf{s} = \mathcal{T}_\kappa(\Omega^* \mathbf{P}^* \mathbf{a}^*),$$

  where $\mathbf{a}^*$ is the sparse representation of $\mathbf{x}$ over the dictionary Γ and $\mathcal{T}$ is a threshold transformation with parameter κ.
- Finally, aggregate all the estimates due to the overlap.



Quantitative Evaluation

- 2 standard datasets for evaluation: **DRIVE** and STARE
- Pre-processing by grayscale conversion and histogram equalization
- 8×8 patches
- Training with 25000 patches
- Evaluation metrics:

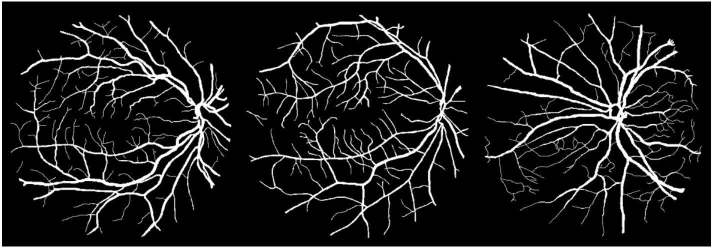
$$\mathbf{Sens} = \frac{TP}{P}, \quad \mathbf{Spec} = \frac{TN}{N}, \quad \mathbf{Acc} = \frac{TP + TN}{P + N}.$$

Methods	**Spec**	**Sens**	**Acc**
This paper	96.78	80.56	**95.34**
Javidi et al.[2]**	97.02	72.01	94.50
SCTC [3]**	95.55	83.49	94.48
Liskowski et al.[5]*	96.73	**84.60**	95.07
Dasgupta et al. [4]*	**98.01**	76.91	95.33

*deep learning methods —— ** dictionary learning methods

Qualitative Results

(a) Predictions



(b) Ground-truths

Conclusion

- We proposed a learning approach for retinal blood vessel segmentation.
- We leverage sparsity as a cue to extract features.
- We discussed a variant where we learned feature extraction and a linear classifier jointly.
- We introduced another variant where we learned data and label dictionaries and a coupling linear mapping.

Thank you for your attention!